By H. G. Miles & L. W. F. Elen

WORKSHOP MATHEMATICS EXERCISES
MOTOR VEHICLE EXERCISES

By L. W. F. Elen & R. Myers

MECHANICS: A NEW INTRODUCTION

By L. W. F. Elen

EXERCISES IN H.N.C. MATHEMATICS

WORKED EXAMPLES IN

Workshop Mathematics

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Preface

THIS book of worked examples is complementary to our book of problems *Workshop Mathematics Exercises*, the call for repeated reprints of which indicates the lively need for material in this field. It is intended to supplement the work of the lecturer and to set out clearly for the student the methods for solving most types of calculation problems encountered in the City and Guilds Intermediate Machine Shop Engineering examination.

The solutions have been put in a form which we think is the easiest to understand, so that in some instances the methods are not necessarily the shortest or most convenient.

Although the examples have been compiled primarily for City and Guilds students, it is hoped that they will be of use to a wider circle concerned with Workshop Mathematics.

We wish to thank the City and Guilds of London Institute for permission to make use of their questions in compiling the collection of miscellaneous examples at the end of the book.

H. G. MILES
L. W. F. ELEN

April 1960

Contents

1. Fractions: Addition and Subtraction

(1) *Find the L.C.M. of* 15, 18, 24.

The L.C.M. of a given set of numbers is the smallest number that can be divided without remainder by all the numbers of the given set. We thus express all the numbers in their prime factors.

$$15 = 3 \times 5$$
$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3$$

Any multiple of 15 must contain 3 and 5, any multiple of 18 must contain 2, 3 and 3, and any multiple of 24 must contain 2, 2, 2 and 3. The smallest set of factors containing all these is 3, 5, 2, 3, 2, 2,

$$\text{L.C.M.} = 3 \times 5 \times 2 \times 3 \times 2 \times 2 = \underline{360}$$

(2) *Arrange in ascending order the following drills:* $\frac{3}{4}$ in., $\frac{7}{8}$ in., $\frac{13}{16}$ in., $\frac{27}{32}$ in., $\frac{23}{32}$ in.

The sizes are compared by arranging each fraction over the L.C.M. of 4, 8, 16, 32, i.e. 32.

$$\frac{3}{4} = \frac{24}{32};\quad \frac{7}{8} = \frac{28}{32};\quad \frac{13}{16} = \frac{26}{32};\quad \frac{27}{32};\quad \frac{23}{32}$$

Ascending order is $\frac{23}{32}$, $\frac{24}{32}$, $\frac{26}{32}$, $\frac{27}{32}$, $\frac{28}{32}$

Required order is $\underline{\frac{23}{32}\text{ in., }\frac{3}{4}\text{ in., }\frac{13}{16}\text{ in., }\frac{27}{32}\text{ in., }\frac{7}{8}\text{ in.}}$

(3) *How much error is involved in taking* $\frac{39}{64}$ *in. as* $\frac{3}{5}$ *in.?*

$$\text{Error} = \frac{39}{64} - \frac{3}{5} = \frac{195 - 192}{320} = \underline{\frac{3}{320}\text{ in.}}$$

(4) *Simplify:* (*a*) $1\frac{1}{4} + \frac{1}{2} + \frac{3}{8}$; (*b*) $2\frac{1}{16} - 1\frac{3}{10}$;
(*c*) $1\frac{5}{8} - \frac{3}{5} + 3\frac{1}{16}$.

$$(a)\ 1\tfrac{1}{4} + \tfrac{1}{2} + \tfrac{3}{8} \quad = 1\frac{2 + 4 + 3}{8} = \underline{2\tfrac{1}{8}}$$

$$(b)\ 2\tfrac{1}{16} - 1\tfrac{3}{10} = 1\frac{5-24}{80} = \frac{80+5-24}{80} = \underline{\tfrac{61}{80}}$$

$$(c)\ 1\tfrac{5}{8} - \tfrac{3}{5} + 3\tfrac{1}{16} = 4\frac{50-48+5}{80} = \underline{4\tfrac{7}{80}}$$

2. Fractions: Multiplication and Division

(1) *Simplify:* (*a*) $\frac{3}{8} \times \frac{2}{3} \times 1\frac{1}{4}$; (*b*) $\frac{3}{8} \div \frac{15}{16}$; (*c*) $8\frac{1}{4} \div 3\frac{1}{7}$.

$$(a)\ \frac{3}{8}\times\frac{2}{3}\times 1\tfrac{1}{4} = \frac{\cancel{3}}{8}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{5}{\underset{2}{\cancel{4}}} = \underline{\frac{5}{16}}$$

$$(b)\ \frac{3}{8}\div\frac{15}{16} = \frac{\cancel{3}}{\cancel{8}}\times\frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{15}}} = \underline{\frac{2}{5}}$$

$$(c)\ 8\tfrac{1}{4}\div 3\tfrac{1}{7} = \frac{\overset{3}{\cancel{33}}}{4}\times\frac{7}{\underset{2}{\cancel{22}}} = \frac{21}{8} = \underline{2\tfrac{5}{8}}$$

(2) *If a tool advances* $\frac{5}{32}$ *in. per revolution, how long will it take for the tool to advance* $7\frac{1}{2}$ *in., if the job is turning at* 100 *rev./min?*

No. of revolutions to advance $7\frac{1}{2}$ in. $= 7\frac{1}{2} \div \frac{5}{32}$

$$= \frac{\overset{3}{\cancel{15}}}{\cancel{2}}\times\frac{\overset{16}{\cancel{32}}}{\cancel{5}} = 48$$

$$\text{Time taken} = \frac{48}{100}\text{ min} = \frac{48\times\overset{3}{\cancel{60}}}{\underset{5}{\cancel{100}}}\text{ sec}$$

$$= \frac{144}{5} = \underline{28\tfrac{4}{5}\text{ sec}}$$

(3) *Equally-spaced holes are drilled in a bar 4 ft 3 in. long as shown in fig. 1. Calculate the number of holes.*

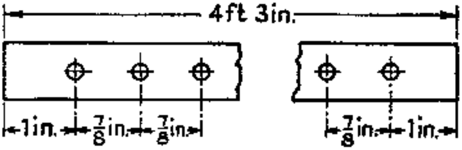


Fig. 1

Distance between first and last hole = 4 ft 1 in. = 49 in.
No. of spaces $= 49 \div \frac{7}{8} = 49 \times \frac{8}{7} = 56$
No. of holes $= 56 + 1 = \underline{57}$

3. Fractions: Problems

(1) *Calculate the dimensions D and d shown in fig. 2.*

$$\begin{aligned} d &= 2\tfrac{5}{8} - \tfrac{1}{2}(1\tfrac{1}{4} + \tfrac{13}{16}) \\ &= 2\tfrac{5}{8} - (\tfrac{1}{2} \times 2\tfrac{1}{16}) \\ &= 2\tfrac{5}{8} - (\tfrac{1}{2} \times \tfrac{33}{16}) \\ &= 2\tfrac{5}{8} - \tfrac{33}{32} \\ &= 2\tfrac{5}{8} - 1\tfrac{1}{32} \\ &= 1\frac{20 - 1}{32} \\ &= \underline{1\tfrac{29}{32} \text{ in.}} \end{aligned}$$

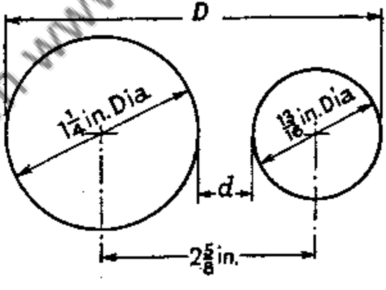


Fig. 2

$$\begin{aligned} D &= 2\tfrac{5}{8} + \tfrac{1}{2}(1\tfrac{1}{4} + \tfrac{13}{16}) \\ &= 2\tfrac{5}{8} + 1\tfrac{1}{32} \\ &= 3\frac{20 + 1}{32} \\ &= \underline{3\tfrac{21}{32} \text{ in.}} \end{aligned}$$

(2) *Allowing $\frac{3}{64}$ in. on the ends of each pin for facing and $\frac{3}{16}$ in. for cutting-off, calculate the length of bar required to make 7 pins as shown in fig. 3.*

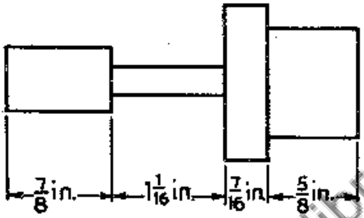


Fig. 3

$$\text{Length of pin} = \tfrac{7}{8} + 1\tfrac{1}{16} + \tfrac{7}{16} + \tfrac{5}{8}$$

$$= 1\frac{14 + 1 + 7 + 10}{16}$$

$$= 1\tfrac{32}{16}$$

$$= 3 \text{ in.}$$

$$\text{Length before facing} = 3 + (2 \times \tfrac{3}{64})$$

$$= 3 + \tfrac{3}{32} = 3\tfrac{3}{32}$$

$$\text{Number of cuts} = 7 - 1 = 6$$

$$\text{Total length} = (7 \times 3\tfrac{3}{32}) + (6 \times \tfrac{3}{16})$$

$$= (7 \times \tfrac{99}{32}) + (\tfrac{6}{1} \times \tfrac{3}{16})$$

$$= \frac{693}{32} + \frac{18}{16}$$

$$= 21\tfrac{21}{32} + 1\tfrac{1}{8}$$

$$= 22\frac{21 + 4}{32}$$

$$= \underline{22\tfrac{25}{32} \text{ in.}}$$

(3) *In fig. 4, calculate the dimensions a and b.*

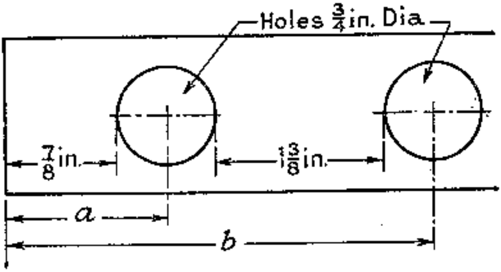


Fig. 4

$$a = \tfrac{7}{8} + (\tfrac{1}{2} \times \tfrac{3}{4}) = \tfrac{7}{8} + \tfrac{3}{8} \qquad = \tfrac{10}{8} = 1\tfrac{1}{4}\text{ in.}$$

$$b = \tfrac{7}{8} + \tfrac{3}{4} + 1\tfrac{3}{8} + (\tfrac{1}{2} \times \tfrac{3}{4}) \qquad = \tfrac{7}{8} + \tfrac{3}{4} + 1\tfrac{3}{8} + \tfrac{3}{8}$$

$$= 1\frac{7 + 6 + 3 + 3}{8}$$

$$= 1\tfrac{19}{8} = \underline{3\tfrac{3}{8}\text{ in.}}$$

4–6. Decimals: Addition, Subtraction, Multiplication, Division and Problems

(1) *Calculate the dimensions D and d shown in fig. 5.*

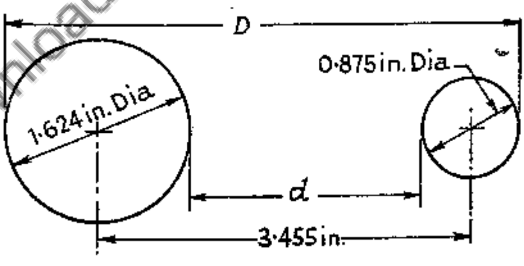


Fig. 5

$$\begin{aligned} D &= 3{\cdot}4550 + \tfrac{1}{2}(1{\cdot}624 + 0{\cdot}875) \\ &= 3{\cdot}4550 + \tfrac{1}{2}(2{\cdot}499) \\ &= 3{\cdot}4550 + 1{\cdot}2495 \\ &= \underline{4{\cdot}7045 \text{ in.}} \end{aligned}$$

$$\begin{aligned} d &= 3{\cdot}4550 - \tfrac{1}{2}(1{\cdot}624 + 0{\cdot}875) \\ &= 3{\cdot}4550 - 1{\cdot}2495 \\ &= \underline{2{\cdot}2055 \text{ in.}} \end{aligned}$$

(2) *Calculate the greatest and least permissible values for the dimension marked x in the fig.* 6.

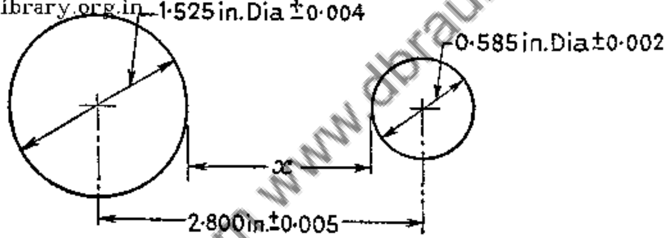


Fig. 6

$$\begin{aligned} \text{Nominal value of } x &= 2{\cdot}800 - \tfrac{1}{2}(1{\cdot}525 + 0{\cdot}585) \\ &= 2{\cdot}800 - \tfrac{1}{2} \times 2{\cdot}110 \\ &= 2{\cdot}800 - 1{\cdot}055 \\ &= \underline{1{\cdot}745 \text{ in.}} \end{aligned}$$

$$\begin{aligned} \text{Greatest value of } x &= 1{\cdot}745 + 0{\cdot}005 + \tfrac{1}{2}(0{\cdot}004 + 0{\cdot}002) \\ &= 1{\cdot}745 + 0{\cdot}005 + 0{\cdot}003 \\ &= \underline{1{\cdot}753 \text{ in.}} \end{aligned}$$

$$\begin{aligned} \text{Least value of } x &= 1{\cdot}745 - 0{\cdot}005 - \tfrac{1}{2}(0{\cdot}004 + 0{\cdot}002) \\ &= 1{\cdot}745 - 0{\cdot}005 - 0{\cdot}003 \\ &= \underline{1{\cdot}737 \text{ in.}} \end{aligned}$$

(3) *Find the greatest and least values of the dimension marked x in fig. 7.*

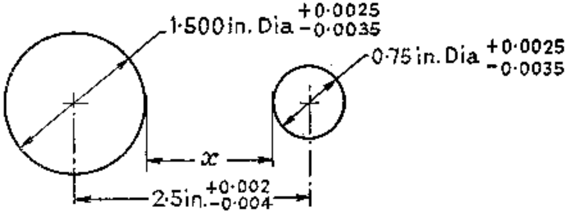


Fig. 7

$$\begin{aligned}
\text{Nominal value of } x &= 2{\cdot}5 - \tfrac{1}{2}(1{\cdot}5 + 0{\cdot}75)\\
&= 2{\cdot}5 - \tfrac{1}{2} \times 2{\cdot}25\\
&= 2{\cdot}5 - 1{\cdot}125\\
&= \underline{1{\cdot}375\text{ in.}}\\
\text{Greatest value of } x &= 1{\cdot}3750 + 0{\cdot}0020 + \tfrac{1}{2}(0{\cdot}0035 + 0{\cdot}0035)\\
&= 1{\cdot}3750 + 0{\cdot}0020 + 0{\cdot}0035\\
&= \underline{1{\cdot}3805\text{ in.}}\\
\text{Least value of } x &= 1{\cdot}375 - 0{\cdot}004 - \tfrac{1}{2}(0{\cdot}0025 + 0{\cdot}0025)\\
&= 1{\cdot}3750 - 0{\cdot}0040 - 0{\cdot}0025\\
&= 1{\cdot}3750 - 0{\cdot}0065\\
&= \underline{1{\cdot}3685\text{ in.}}
\end{aligned}$$

NOTE. When correcting to a given number of decimal places, if the next figure is 5 or above, add 1, but if it is 4 or less, ignore it; e.g.

1·5076	correct to	3	decimal	places	1·508
1·2743	,,	,,	,,	,,	1·274
1·86347	,,	,,	,,	,,	1·863

The same rule is applied when correcting to a given number of significant figures.

(4) *Give:* (*a*) 14·235 × 2·425; (*b*) 2·473 × 0·0625 *correct to 4 decimal places.*

METHOD. Multiply as in ordinary multiplication, taking no notice of the decimal points. Then mark off in the product as many decimal places as there are in both the original numbers together. Thus:

$$
(a)\quad
\begin{array}{r}
14{\cdot}235 \\
2{\cdot}425 \\
\hline
71\,175 \\
284\,70 \\
5\,694\,0 \\
28\,470 \\
\hline
34{\cdot}519\,875 \\
\\
\underline{34{\cdot}5199}
\end{array}
\qquad\qquad
(b)\quad
\begin{array}{r}
2{\cdot}473 \\
0{\cdot}00625 \\
\hline
1\,2365 \\
4\,946 \\
148\,38 \\
\hline
0{\cdot}0154\,5625 \\
\\
\underline{0{\cdot}0155}
\end{array}
$$

(5) *Give:* (a) 13·75 ÷ 2·64; (b) 0·0638 ÷ 0·875 *correct to 4 decimal places.*

METHOD. Make the divisor a whole number by writing the division sum as a fraction so that one decimal point is directly under the other and by drawing a vertical line behind the last digit of the denominator. Thus:

$$
(a)\ \left.\frac{13{\cdot}75}{2{\cdot}64}\right| \qquad\qquad (b)\ \frac{0{\cdot}063}{0{\cdot}875}\left|\frac{8}{}\right.
$$

$$
\begin{array}{r}
\overline{5{\cdot}20833} \\
264)1375{\cdot} \\
1320\phantom{{\cdot}833} \\
\hline
55\,0 \\
52\,8 \\
\hline
2\,200 \\
2\,112 \\
\hline
880 \\
792 \\
\hline
880 \\
792 \\
\hline
88 \\
\\
\underline{5{\cdot}2083}
\end{array}
\qquad\qquad
\begin{array}{r}
\overline{0{\cdot}07291} \\
875)63{\cdot}80 \\
61\,25 \\
\hline
2\,550 \\
1\,750 \\
\hline
8000 \\
7875 \\
\hline
1250 \\
875 \\
\hline
375 \\
\\
\underline{0{\cdot}0729}
\end{array}
$$

7. Decimals and Fractions: Conversion

(1) *Convert to decimals:* (*a*) $\frac{3}{8}$; (*b*) $\frac{11}{12}$; (*c*) $\frac{23}{64}$.

METHOD. Divide the numerator by the denominator. Thus:

$$(a)\quad \frac{3}{8} = \begin{array}{r|l} 8 & \underline{3{\cdot}000} \\ & 0{\cdot}375 \end{array}$$

$$= \underline{0{\cdot}375}$$

$$(b)\quad \frac{11}{12} = \begin{array}{r|l} 12 & \underline{11{\cdot}00000} \\ & 0{\cdot}91666 \end{array}$$

$$= \underline{0{\cdot}9167} \text{ correct to 4 decimal places}$$

$$(c)\quad \frac{23}{64} = \begin{array}{r l} & 0{\cdot}35937 \\ 64) & 23{\cdot}0000 \\ & \underline{19\ 2} \\ & 3\ 80 \\ & \underline{3\ 20} \\ & 600, \text{ etc.} \end{array}$$

$$= \underline{0{\cdot}3594} \text{ correct to 4 decimal places}$$

(2) *Convert the following decimals to fractions in their lowest form:*

(*a*) 0·725; (*b*) 2·048; (*c*) 1·84.

METHOD. Put decimal part as the numerator of a fraction, the denominator being 1 followed by as many zeros as there are decimal places in the original decimal.

$$(a)\quad 0{\cdot}725 = \frac{725}{1000} = \underline{\frac{29}{40}}$$

$$(b)\quad 2{\cdot}048 = 2\frac{48}{1000} = \underline{2\frac{6}{125}}$$

$$(c)\quad 1{\cdot}84 = 1\frac{84}{100} = \underline{1\frac{21}{25}}$$

(3) *A mechanic wishing to measure* 0·845 *in. has only a ruler marked in* $\frac{1}{32}$ *in. What is the nearest he can make it and what will be his error?*

$$\frac{1}{32} = 0{\cdot}03125$$

$$\text{No. of } \frac{1}{32} = \frac{0{\cdot}845}{0{\cdot}03125} = \frac{84500}{3125}$$

$$\begin{array}{r} 27 \\ 3125\overline{)84500} \\ 6250 \\ \hline 22000 \\ 21875 \\ \hline 1250 \end{array}$$

The remainder 1250 is less than half of 3125.

$$\therefore \text{ nearest} = \underline{\tfrac{27}{32}} \text{ and error} = \underline{0{\cdot}0013 \text{ in.}}$$

(the remainder)

(4) *Convert* $1\frac{3}{8}$ *in. to a decimal of* 1 *ft.*

$1\frac{3}{8}$ in. as a fraction of 1 ft.

$$= 1\tfrac{3}{8} \div 12$$

$$= \frac{11}{8} \times \frac{1}{12}$$

$$= \frac{11}{96}$$

$1\frac{3}{8}$ in. as a decimal of 1 ft.

$$\begin{array}{r} 0{\cdot}1145 \\ = 96\overline{)11{\cdot}0000} \\ 96 \\ \hline 140 \\ 96 \\ \hline 440 \\ 384 \\ \hline 560 \end{array}$$

$$\therefore 1\tfrac{3}{8} \text{ in.} = \underline{0{\cdot}115 \text{ ft.}}$$

8–12. Logarithms: Positive and Negative Characteristics. Multiplication, Division, Powers and Roots

Calculations can be made much easier if logarithms are used instead of employing long-hand methods.

A logarithm of a number consists of 2 parts:

(i) the whole number part (or characteristic). This indicates the position of the decimal point.

(ii) the decimal part (or mantissa). This is found from tables of logarithms.

Below is part of a table of logarithms.

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
46	·6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	·6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	·6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	·6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	·6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

FOR NUMBERS GREATER THAN 1

Characteristic = number of figures before the decimal point minus 1

(1) *To find log* 46·2.

The characteristic is 2 − 1 = 1 and the decimal part is found by looking along the line at 46 in the log tables and under the 2 is found 0·6646 (the decimal point is usually omitted except at the beginning of the line).

$$\text{Thus, } \log 46{\cdot}2 = \underline{1{\cdot}6646}$$

(2) *To find log* 4768.

The characteristic is 4 − 1 = 3. To find the decimal part, look along the row 47 in the log tables and under 6 is found 0·6776. To obtain the last figure we go to the end columns and under the 8 is found a 7 which is added to 0·6776 to give 0·6783.

$$\therefore \log 4768 = 3{\cdot}6783$$

ANTILOGARITHM TABLES

After using logarithms, it is necessary to revert to numbers. For this, tables of antilogarithms are used.

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7

METHOD. (*a*) Look up the decimal part only, by the same method as was used for finding logarithms.

(*b*) The position of the decimal point is found by the rule: Number of figures before the decimal point = characteristic + 1. Thus:

Antilog	Number
0·4725	2·968
2·4725	296·8
1·4725	29·68

NOTE. Answers obtained using four-figure logarithm tables are not necessarily accurate to four significant figures. There may be an error in the last figure. If four-figure accuracy is required, the calculation must be carried out either by using five-figure logarithm tables or by long-hand methods.

Four useful processes can be carried out by using logarithms.

(i) Multiplication: add the logarithms of the numbers and look up the antilogarithm of the result.

(ii) Division: subtract the logarithms of the numbers and look up the antilogarithm of the result.

(iii) Powers: multiply the logarithm of the number by the power and look up the antilogarithm.

(iv) Roots: divide the logarithm of the number by the root and look up the antilogarithm.

(3) *Using logarithms find:*

22·37 × 1·965.

= 43·95

No.	Log
22·37	1·3496
1·965	0·2934
	1·6430 *add*

antilog = 43·95

(4) *Using logarithms find:*

89·52 ÷ 11·45.

= <u>7·818</u>

No.	Log	
89·52	1·9519	
11·45	1·0588	
	0·8931	*subtract*

antilog = 7·818

(5) *Using logarithms find:*

12·24 × 1·735 × 10·06.

= <u>213·5</u>

No.	Log	
12·24	1·0878	
1·735	0·2392	
10·06	1·0025	
	2·3295	*add*

antilog = 213·5

(6) *Using logarithms find:*

$$\frac{17\cdot49 \times 52\cdot10}{6\cdot295}.$$

= <u>144·7</u>

No.	Log	
17·49	1·2427	
52·10	1·7168	
	2·9595	*add*
6·295	0·7990	
	2·1605	*subtract*

antilog = 144·7

(7) *Using logarithms find:*

$$\frac{6\cdot72 \times 45\cdot75}{2\cdot795 \times 3\cdot142}$$

= <u>35·01</u>

No.	Log	
2·795	0·4464	
3·142	0·4972	
Denominator	0·9436	*add*
6·72	0·8274	
45·75	1·6604	
Numerator	2·4878	*add*
Denominator	0·9436	
	1·5442	*subtract*

antilog = 35·01

(8) *Find by logarithms:*

$7{\cdot}375^2$.

$= \underline{54{\cdot}41}$

No.	Log
7·375	0·8678
multiply by	2
	1·7356

antilog = 54·41

(9) *Find by logarithms:*

$\left(\frac{11{\cdot}42}{3{\cdot}745}\right)^3$.

$= \underline{28{\cdot}35}$

No.	Log	
11·42	1·0577	
3·745	0·5735	*subtract*
	0·4842	
multiply by	3	
	1·4526	

antilog = 28·35

(10) *Find by logarithms:*

$\sqrt{11{\cdot}75}$.

$\underline{3{\cdot}428}$

No.	Log
11·75	1·0700
divide by 2	0·5350

antilog = 3·428

(11) *Find by logarithms:*

$\sqrt[3]{\left(\frac{49{\cdot}46}{\pi}\right)}$.

$= 2{\cdot}508$

No.	Log	
49·56	1·6951	
π	0·4971	*subtract*
divide by 3	1·1980	
	0·3993	

antilog = 2·508

NUMBERS LESS THAN 1—NEGATIVE CHARACTERISTICS

When a number is less than 1, the characteristic is negative. As the mantissa (decimal part of the logarithm) is still positive, the normal minus sign cannot be used. Therefore a line, or bar, is written over the characteristic, thus indicating that the characteristic is negative but the decimal part is still positive, e.g.

$\bar{1}{\cdot}732$ means $-1 + 0{\cdot}732$

The numerical value of the characteristic is found by the rule: characteristic is one more than the number of zeros between the decimal point and the first figure that is not zero. Thus:

Number	Log
0·065	$\bar{2}$·8129
0·739	$\bar{1}$·8686
0·0005	$\bar{4}$·6990

When finding the antilogarithm of a 'bar' number, use the antilogarithm tables as before. The position of the decimal point is found by the following rule:

No. of zeros between the decimal point and the first figure that is not zero is one less than the characteristic.

NEGATIVE NUMBERS

If +2 represents two steps forward, then −2 represents two steps backwards. Thus, +2 − 3 means 2 steps forward, followed by 3 steps backwards. This is the same as 1 step backwards:

$$2 - 3 = -1. \text{ Similarly } 5 - 8 = -3.$$

Also, −2 − 3 = −5 since it means 2 steps backwards followed by 3 steps backwards.

Since a minus sign signifies reversed direction, then

$$-(-2) = +2$$

because two reversals brings us back to the original direction. In logarithms, the minus sign is placed over the top of the number.

Thus:

$$\bar{2} + \bar{1} = (-2) + (-1) = -2 - 1 = -3 = \bar{3}$$
$$1 + \bar{3} = 1 + (-3) = 1 - 3 = -2 = \bar{2}$$
$$\bar{3} - \bar{2} = (-3) - (-2) = -3 + 2 = -1 = \bar{1}$$
$$-\bar{1} + \bar{3} = -(-1) + (-3) = 1 - 3 = -2 = \bar{2}$$

(12) *Add:* (*a*) $\bar{1}\cdot 4 + \bar{2}\cdot 7$; (*b*) $\bar{3}\cdot 8 + 1\cdot 7$.

(*a*) $\bar{1}\cdot 4 + \bar{2}\cdot 7 = -1 + 0\cdot 4 - 2 + 0\cdot 7 = -3 + 1\cdot 1$

$$= -3 + 1 + 0\cdot 1 = \underline{\bar{2}\cdot 1}$$

It should not of course be necessary to work the questions as fully as this.

(*b*) $\bar{3}\cdot 8 + 1\cdot 7 = \underline{\bar{1}\cdot 5}$.

(13) *Subtract:* (*a*) $\bar{2}\cdot 7$ *from* $\bar{1}\cdot 5$; (*b*) $\bar{3}\cdot 8$ *from* $\bar{1}\cdot 7$.

$$\begin{aligned} (a)\ \bar{1}\cdot 5 - \bar{2}\cdot 7 &= -1 + 0\cdot 5 - (-2 + 0\cdot 7) \\ &= -1 + 0\cdot 5 + 2 - 0\cdot 7 \\ &= 1 - 0\cdot 2 = \underline{0\cdot 8} \end{aligned}$$

$$\begin{aligned} (b)\ \bar{1}\cdot 7 - \bar{3}\cdot 8 &= -1 + 0\cdot 7 - (-3 + 0\cdot 8) \\ &= -1 + 0\cdot 7 + 3 - 0\cdot 8 \\ &= \underline{1\cdot 9} \end{aligned}$$

(14) *Multiply:* (*a*) $\bar{1}\cdot 7 \times 2$; (*b*) $\bar{2}\cdot 6 \times 3$.

$$\begin{aligned} (a)\ 2(\bar{1}\cdot 7) &= 2(-1 + 0\cdot 7) \\ &= -2 + 1\cdot 4 = \underline{\bar{1}\cdot 4} \end{aligned}$$

$$\begin{aligned} (b)\ 3(\bar{2}\cdot 6) &= 3(-2 + 0\cdot 6) \\ &= -6 + 1\cdot 8 = \underline{\bar{5}\cdot 8} \end{aligned}$$

(15) *Divide:* (*a*) $\bar{1}\cdot 6$ *by* 2; (*b*) $\bar{2}\cdot 5$ *by* 3.

$$\begin{aligned} (a)\ \tfrac{1}{2}(\bar{1}\cdot 6) &= \tfrac{1}{2}(-1 + 0\cdot 6) \\ &= \tfrac{1}{2}(-2 + 1\cdot 6) = \underline{\bar{1}\cdot 8} \end{aligned}$$

$$\begin{aligned} (b)\ \tfrac{1}{3}(\bar{2}\cdot 5) &= \tfrac{1}{3}(-2 + 0\cdot 5) \\ &= \tfrac{1}{3}(-3 + 1\cdot 5) = \underline{\bar{1}\cdot 5} \end{aligned}$$

(16) *Multiply:*

$0\cdot 0365 \times 0\cdot 2467$.

$= \underline{0\cdot 009005}$

No.	Log	
0·0365	$\bar{2}\cdot 5623$	
0·2467	$\bar{1}\cdot 3922$	*add*
	$\bar{3}\cdot 9545$	

antilog = 0·009005

(17) *Divide:*

$0\cdot 0365 \div 0\cdot 2467$.

$= \underline{0\cdot 1479}$

No.	Log	
0·0365	$\bar{2}\cdot 5623$	
0·2467	$\bar{1}\cdot 3922$	*subtract*
	$\bar{1}\cdot 1701$	

antilog = 0·1479

(18) *Divide:*

$$29\cdot57 \div 119\cdot8.$$

$= \underline{0\cdot2468}$

No.	Log	
29·57	1·4708	
119·8	2·0785	*subtract*
	$\bar{1}\cdot3923$	

antilog = 0·2468

(19) *Divide:*

$$0\cdot3862 \div 763\cdot2.$$

$= \underline{0\cdot0005060}$

No.	Log	
0·3862	$\bar{1}\cdot5868$	
763·2	2·8826	*subtract*
	$\bar{4}\cdot7042$	

antilog = 0·0005060

(20) *Evaluate:*

$$\frac{2\cdot647 \times 0\cdot00921}{0\cdot0005738}.$$

$= \underline{42\cdot48}$

No.	Log	
2·647	0·4227	
0·00921	$\bar{3}\cdot9643$	*add*
	$\bar{2}\cdot3870$	
0·0005738	$\bar{4}\cdot7588$	*subtract*
	1·6282	

antilog = 42·48

(21) *Evaluate:*

$$\frac{0\cdot3826^2}{0\cdot5985^3}.$$

$= \underline{0\cdot6828}$

No.	Log	
0·5985	$\bar{1}\cdot7771$	
	3	*multiply*
denominator	$\bar{1}\cdot3313$	
0·3826	$\bar{1}\cdot5828$	
	2	*multiply*
numerator	$\bar{1}\cdot1656$	
denominator	$\bar{1}\cdot3313$	
	$\bar{1}\cdot8343$	

antilog = 0·6828

(22) *Find the square root of:*

0·02579.

= 0·1606

No.	Log
0·02579	$\bar{2}$·4114
divide by 2	$\bar{1}$·2057

antilog = 0·1606

(23) *Find the square root of:*

0·2579.

= 0·5078

No.	Log
0·2579	$\bar{1}$·4114
divide by 2	$\bar{1}$·7057

antilog = 0·5078

(24) *Solve:*

$$6{\cdot}283\sqrt{\left(\frac{8{\cdot}971}{95{\cdot}32}\right)}.$$

= 1·928

No.	Log	
8·971	0·9528	
95·32	1·9792	*subtract*
divide by 2	$\bar{2}$·9736	
	$\bar{1}$·4868	
6·283	0·7982	*add*
	0·2850	

antilog = 1·928

(25) *Divide:*

$$\frac{1}{15{\cdot}75}.$$

= 0·06349

No.	Log	
1	0·0000	
15·75	1·1973	*subtract*
	$\bar{2}$·8027	

antilog = 0·06349

(26) *Divide:*

$$\frac{1}{0{\cdot}3575}.$$

= 2·797

No.	Log	
1	0·0000	
0·3575	$\bar{1}$·5533	*subtract*
	0·4467	

antilog = 2·797

(27) *Solve:*

$$\sqrt[3]{\left(\frac{15{\cdot}5 \times 0{\cdot}705}{41{\cdot}5}\right)}.$$

$$= \underline{0{\cdot}6409}$$

No.	Log	
15·5	1·1903	
0·705	$\bar{1}$·8482	*add*
	1·0385	
41·5	1·6180	*subtract*
	$\bar{1}$·4205	
divide by 3	$\bar{1}$·8068	

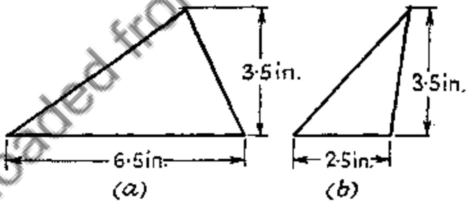
antilog = 0·6409

13. Mensuration: Triangles and Rectangles

FORMULAE

Area of a triangle = $\frac{1}{2}$ × base × altitude
Area of a rectangle = length × breadth

(1) *Calculate the areas of the triangles shown in fig.* 8.



(*a*) (*b*)

Fig. 8

$$(a)\ \text{Area} = \tfrac{1}{2} \times \text{base} \times \text{altitude}$$
$$= \tfrac{1}{2} \times 6{\cdot}5 \times 3{\cdot}5$$
$$= \underline{11{\cdot}38 \text{ in.}^2}$$

$$(b)\ \text{Area} = \tfrac{1}{2} \times 2{\cdot}5 \times 3{\cdot}5$$
$$= \underline{4{\cdot}38 \text{ in.}^2}$$

(2) *Calculate the base of a triangle having an area of* 15 *in.*^{2} *if its altitude is* 5·25 *in.*

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{altitude}$$
$$15 = \tfrac{1}{2} \times \text{base} \times 5{\cdot}25$$
$$\text{base} = \frac{15 \times 2}{5{\cdot}25}$$
$$= \underline{5{\cdot}714 \text{ in.}}$$

(3) *Calculate the area of the section shown in fig.* 9. *If it is made of sheet metal weighing* 4·5 *lb/in.*^{2}, *calculate its weight.*

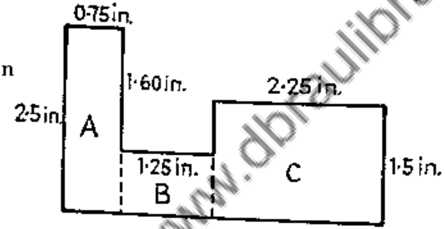


Fig. 9

$$\text{Area of } A = 2{\cdot}5 \times 0{\cdot}75 = 1{\cdot}875 \text{ in.}^2$$
$$\text{Area of } B = 1{\cdot}25 \times 0{\cdot}9 = 1{\cdot}125 \text{ in.}^2$$
$$\text{Area of } C = 2{\cdot}25 \times 1{\cdot}5 = 3{\cdot}375 \text{ in.}^2$$
$$\text{Total Area} = 6{\cdot}375 \text{ in.}^2$$

$$\text{Weight} = \frac{6{\cdot}375}{144} \times \frac{4{\cdot}5}{1}$$
$$= 0{\cdot}199 \text{ lb}$$
$$= \underline{3{\cdot}14 \text{ oz.}}$$

(4) *Calculate the length of strip metal* 2 *in. wide if it weighs* 7·5 *lb and is made from strip weighing* 5·25 *lb/ft.*^{2}

$$\text{Area of metal} = \frac{7{\cdot}5}{5{\cdot}25} \text{ ft}^2 = \frac{7{\cdot}5}{5{\cdot}25} \times 144 \text{ in.}^2$$
$$= 205{\cdot}7 \text{ in.}^2$$

$$\text{Length of strip} = \frac{\text{area}}{\text{width}} = \frac{205{\cdot}7}{2} = \underline{102{\cdot}8 \text{ in.}}$$

14. Mensuration: Circles

FORMULAE

If d is the diameter and r is the radius

$$\text{Circumference } (C) = \pi d = 2\pi r$$

$$\text{Area } (A) = \frac{\pi d^2}{4} = \pi r^2$$

Given the circumference, to find the diameter $d = \dfrac{C}{\pi}$

Given the area, to find the diameter $d = \sqrt{\left(\dfrac{4A}{\pi}\right)}$

$\pi = 3{\cdot}142$ or $22/7$ (approx.) Take $\log \pi = 0{\cdot}4971$

(1) *Calculate the circumference and area of a circle whose diameter is* 6·5 *in.*

$$\begin{aligned} \text{Circumference} &= \pi d \\ &= \pi \times 6{\cdot}5 \\ &= \underline{20{\cdot}42 \text{ in.}} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{\pi d^2}{4} \\ &= \frac{\pi \times 6{\cdot}5^2}{4} \\ &= \underline{33{\cdot}17 \text{ in.}^2} \end{aligned}$$

(calculation shown)

No.	Log
6·5	0·8129
	2
	1·6258
π	0·4971
	2·1229
4	0·6021
	1·5208

(2) *Calculate the radius of a circle whose circumference is* 62·5 *in.*

$$\text{Diameter} = \frac{\text{circumference}}{\pi} = \frac{62{\cdot}5}{\pi} = 19{\cdot}90 \text{ in.}$$

$$\therefore \text{ Radius} = \frac{19{\cdot}90}{2} = \underline{9{\cdot}95 \text{ in.}}$$

(3) *Calculate the diameter of a shaft whose cross-sectional area is* 3·565 *in.*^{2}

$$\text{Diameter} = \sqrt{\left(\frac{4A}{\pi}\right)}$$

$$= \sqrt{\left(\frac{4 \times 3{\cdot}565}{\pi}\right)}$$

$$= \underline{2{\cdot}13 \text{ in.}} \text{ (calculation shown)}$$

No.	Log
3·565	0·5520
4	0·6021
	1·1541
π	0·4971
	0·6570
÷ 2	0·3285

(4) *Calculate the area of the shaded portion in fig.* 10.

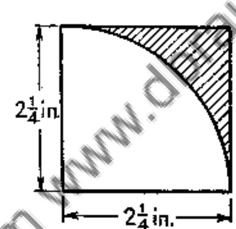


Fig. 10

$$\text{Area of shaded portion} = \text{Area of square} - \text{area of quadrant}$$

$$= 2\tfrac{1}{4}^2 - \frac{\pi \times 2\frac{1}{4}^2}{4}$$

$$= 5{\cdot}063 - 3{\cdot}977$$

$$= \underline{1{\cdot}086 \text{ in.}^2}$$

(5) *Calculate the area of the shaded portion in fig.* 11.

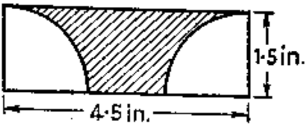


Fig. 11

$$\text{Area of rectangle} = 4{\cdot}5 \times 1{\cdot}5 = 6{\cdot}75 \text{ in.}^2$$

The area of the non-shaded portions is equal to the area of a semicircle of radius 1·5 in.

$$\begin{aligned}\text{Area of the non-shaded portions} &= \pi \times \frac{1{\cdot}5^2}{2}\\ &= 3{\cdot}535 \text{ in.}^2\\ \text{Area of the shaded portion} &= 6{\cdot}750 - 3{\cdot}535\\ &= \underline{3{\cdot}215 \text{ in.}^2}\end{aligned}$$

15. Cutting Speeds; Speeds of Belts

(1) *Calculate the speed of a belt in ft/min passing over a pulley of 18-in. diameter and rotating at 75 rev/min.*

In one revolution the distance travelled by a point on the rim of the pulley $= \pi d = 18\pi$ in.

In 75 revolutions, distance travelled by the point

$$\begin{aligned} &= 18\pi \times 75 \text{ in.}\\ &= \frac{18\pi \times 75}{12} \text{ ft}\\ &= 353 \text{ ft}\\ \text{Speed} &= \underline{353 \text{ ft/min}}\end{aligned}$$

(2) *A belt travelling at 215 ft/min passes over a pulley 16 in. in diameter. Calculate the rev/min of the pulley.*

In one revolution distance travelled by a point on the rim

$$= \pi d = 16\pi \text{ in.}$$

$$\begin{aligned}\text{Number of rev/min} &= \frac{\text{distance travelled/min}}{\text{distance travelled in 1 rev}}\\ &= \frac{215 \times 12}{16\pi}\\ &= \underline{50{\cdot}6}\end{aligned}$$

(3) *If the cutting speed for a given material is* 55 *ft/min and the job is* 8 *in. in diameter, calculate the rev/min required.*

Distance travelled by a point on the rim in 1 rev = 8π in.

$$\begin{aligned}\text{Rev/min} &= \frac{\text{total distance/min}}{\text{distance covered in 1 rev}}\\ &= \frac{55 \times 12}{8\pi}\\ &= \underline{26{\cdot}3}\end{aligned}$$

(4) *An* 8*-in. diameter grinding wheel rotates at* 1500 *rev/min. What is the speed of a point on the surface of the wheel?*

If it rotated 1 rev/min, distance travelled by a point on the rim = 8π in.

For a speed of 1500 rev/min, distance travelled/min will be:

$$\begin{aligned}&8\pi \times 1500 \text{ in.}\\ &= \frac{8\pi \times 1500}{12} \text{ ft}\\ &= \underline{3140 \text{ ft}}\end{aligned}$$

(5) *Calculate the time taken to drill through a plate* $1\frac{1}{4}$ *in. thick using a* $\frac{1}{2}$*-in. drill cutting at* 50 *ft/min, the feed being* 0·03 *in./rev.*

$$\begin{aligned}\text{No. of rev/min of drill} &= \frac{50 \times 12}{\pi \times 0{\cdot}5}\\ &= \frac{600}{0{\cdot}5\pi}\\ &= 382\end{aligned}$$

$$\text{Feed per min} = 382 \times 0{\cdot}03 \quad = 11{\cdot}46 \text{ in.}$$

$$\text{Time} = \frac{1{\cdot}25}{11{\cdot}46} \times 60 \quad = \underline{6{\cdot}5 \text{ sec}}$$

16. Percentages

NOTE. To express a number as a percentage, multiply the number by 100.

(1) *Express* $\frac{7}{15}$ *as a percentage.*

$$\text{Percentage} = \frac{7}{15} \times 100 = \frac{140}{3} = \underline{46{\cdot}67}$$

NOTE. To express a percentage as an ordinary number divide by 100.

(2) *Express* 85 *per cent as a fraction.*

$$85 \text{ per cent} = \frac{85}{100} = \underline{\frac{17}{20}}$$

(3) *Calculate* 35 *per cent of* 165.

$$35 \text{ per cent of } 165 = 165 \times \frac{35}{100} = \underline{57{\cdot}8}$$

(4) *What percentage of* 180 *is* 85?

$$\text{Percentage} = \frac{85}{180} \times \frac{100}{1} = \underline{47{\cdot}2}$$

(5) *The length of a pin was found to be* 2·063 *in. The specification called for a length of* 2·075 *in. Calculate the percentage error.*

$$\text{Error} = 2{\cdot}075 - 2{\cdot}063 = 0{\cdot}012 \text{ in.}$$

$$\text{Percentage error} = \frac{0{\cdot}012}{2{\cdot}075} \times \frac{100}{1} = \underline{0{\cdot}58}$$

(6) *The weight of a casting was given as* 125 *lb with a possible error of* 1·5 *per cent. Calculate the permissible range of weights.*

$$1{\cdot}5 \text{ per cent of } 125 = 125 \times \frac{1{\cdot}5}{100} = 1{\cdot}875$$

$$\begin{aligned}\text{Range} &= 125 \pm 1{\cdot}875\\ &= \underline{123{\cdot}125 \text{ lb to } 126{\cdot}875 \text{ lb}}\end{aligned}$$

(7) *Plates in the form of a quadrant of a circle of 2-in. radius are stamped out of squares of side $2\frac{1}{4}$ in. Calculate the percentage of the metal scrapped.*

$$\text{Area of square} = 2\tfrac{1}{4} \times 2\tfrac{1}{4} = 5\cdot063 \text{ in.}^2$$

$$\text{Area of quadrant} = \frac{\pi}{4} \times 2^2 = 3\cdot142 \text{ in.}^2$$

$$\text{Area scrapped} = 1\cdot921 \text{ in.}^2$$

$$\text{Percentage scrapped} = \frac{1\cdot921}{5\cdot063} \times \frac{100}{1} = \underline{37\cdot9}$$

17. Ratio and Proportion. Percentages

(1) *Divide a length of* 7·5 *in. into three parts in the ratio* 3 : 4 : 5.

Total number of parts = 3 + 4 + 5 = 12

Fractions into which length is divided are $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$

$$\text{Lengths are (i) } 7\cdot5 \times \tfrac{3}{12} = \underline{1\cdot875 \text{ in.}}$$

$$\text{(ii) } 7\cdot5 \times \tfrac{4}{12} = \underline{2\cdot500 \text{ in.}}$$

$$\text{(iii) } 7\cdot5 \times \tfrac{5}{12} = \underline{3\cdot125 \text{ in.}}$$

(2) *An alloy consists of* 61·6 *per cent copper,* 2·9 *per cent lead,* 0·2 *per cent tin and* 35·3 *per cent zinc. Calculate the weight of each in a casting weighing* 240 *lb.*

$$\text{Wt. of copper} = 240 \times \frac{61\cdot6}{100} = \underline{147\cdot84 \text{ lb}}$$

$$\text{Wt. of lead} = 240 \times \frac{2\cdot9}{100} = \underline{6\cdot96 \text{ lb}}$$

$$\text{Wt. of tin} = 240 \times \frac{0\cdot2}{100} = \underline{0\cdot48 \text{ lb}}$$

$$\text{Wt. of zinc} = 240 \times \frac{35\cdot3}{100} = \underline{84\cdot72 \text{ lb}}$$

(3) *If, in Example (2), only 5 lb of lead was available, calculate the weight of alloy of the same composition that could be made.*

5 lb is 2·9 per cent of the total weight

$$\text{Total weight} = \frac{5}{2{\cdot}9} \times 100 = 172 \text{ lb}$$

Only $\underline{172 \text{ lb}}$ of alloy could be made

(4) *How much tin and lead are required to make an alloy consisting of tin, antimony and lead in the ratio* 15 : 4 : 1 *if the weight of antimony available is 5 lb?*

$$\text{Wt. of tin} = 5 \times \frac{15}{4} = \frac{75}{4} = \underline{18{\cdot}75 \text{ lb}}$$

$$\text{Wt. of lead} = 5 \times \tfrac{1}{4} = \underline{1{\cdot}25 \text{ lb}}$$

(5) *The weight of a casting if made of cast iron is 25 lb. Calculate the weight of a similar casting if made of aluminium. (Cast iron weighs* 0·26 *lb/in.*^{3} *and aluminium weighs* 0·09 *lb/in.*^{3}*) What is the percentage increase in cost if aluminium is used instead of cast iron if aluminium costs four times as much as cast iron per lb?*

$$\text{Volume of casting} = \frac{25}{0{\cdot}26} \text{in.}^3$$

$$\text{Weight of casting in aluminium} = \frac{25}{0{\cdot}26} \times 0{\cdot}09 = \underline{8{\cdot}65 \text{ lb}}$$

$$\text{Weight in aluminium} = \frac{0{\cdot}09}{0{\cdot}26} \times \text{weight in cast iron}$$

$$\text{Cost in aluminium} = \frac{0{\cdot}09}{0{\cdot}26} \times 4 \text{ cost in cast iron}$$

$$= 1{\cdot}385 \times \text{cost in cast iron}$$

$$\text{Increase in cost} = 0{\cdot}385 \times \text{cost in cast iron}$$
$$\text{Percentage increase} = 0{\cdot}385 \times 100 = \underline{38{\cdot}5}$$

18. Tapers

A taper can be expressed in several ways:

(i) in inches per ft,
(ii) in ratio form, i.e. 1 in so and so,
(iii) a coded number,
(iv) in degrees—for examples using this form, refer to page 61 (Examples 31).

(1) *A No. 4 Morse taper has a taper of* 0·6233 *in./ft. Express this in the form of a ratio. What would be the taper in* 5·5 *in.?*

$$0{\cdot}6233 \text{ in./ft} = \frac{0{\cdot}6233}{12} \text{ in./in.}$$

$$= 0{\cdot}0519 \text{ in./in.}$$

$$\text{Taper} = 1 \text{ in. in } \frac{1}{0{\cdot}0519} \text{ in.}$$

$$= 1 \text{ in. in } 19{\cdot}27 \text{ in.}$$

$$= \underline{1/19{\cdot}27}$$

$$\text{In } 5{\cdot}5 \text{ in. taper} = 0{\cdot}0519 \times 5{\cdot}5$$

$$= \underline{0{\cdot}2855 \text{ in.}}$$

(2) *Calculate the taper in in./ft of the tapered section shown in fig.* 12. *Calculate also the set-over of the tailstock for turning the taper.*

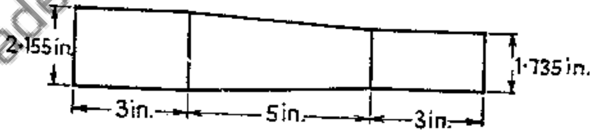


Fig. 12

$$\text{Reduction in dia} = 2{\cdot}155 - 1{\cdot}735$$

$$= 0{\cdot}420 \text{ in.}$$

In 5 in. the reduction is 0·420 in.

In 1 in. „ „ „ $\frac{0{\cdot}420}{5}$ in.

$$\text{In 12 in. the reduction} = \frac{0{\cdot}420}{5} \times \frac{12}{1} = 1{\cdot}008 \text{ in.}$$

$$\text{Taper} = \underline{1{\cdot}008 \text{ in./ft}}$$

For turning the taper by setting-over the tailstock, the total length of the job must be considered as it is impossible to set-over just the centre section.

For any taper turning, the set-over = ½ reduction in dia.

If job were 5-in. long, set-over would be $\frac{0{\cdot}420}{2}$ in.

But since the job is 11-in. long, set-over for whole length

$$= 0{\cdot}210 \times \frac{11}{5}$$

$$= \underline{0{\cdot}462 \text{ in.}}$$

(3) *Calculate the diameter of the workpiece shown in fig. 12 at a distance of 4·5 in. from the larger end.*

$$\text{Reduction in dia/in.} = \frac{0{\cdot}420}{5} = 0{\cdot}084 \text{ in.}$$

For a distance 4·5 in. from larger end, required taper length = 1·5 in.

$$\text{Reduction in diameter} = 0{\cdot}084 \times 1{\cdot}5$$
$$= 0{\cdot}126 \text{ in.}$$

$$\text{Required diameter} = 2{\cdot}155 - 0{\cdot}126$$
$$= \underline{2{\cdot}029 \text{ in.}}$$

(4) *Calculate the distance from the smaller end of the workpiece shown in fig. 12 where the diameter is 2·000 in.*

$$\text{Increase in diameter} = 2{\cdot}000 - 1{\cdot}735$$
$$= 0{\cdot}265 \text{ in.}$$

$$\text{Distance from smaller end} = 3 + \frac{0{\cdot}265}{0{\cdot}084}$$
$$= 3 + 3{\cdot}155$$
$$= \underline{6{\cdot}155 \text{ in.}}$$

19. Metric System

CONVERSION VALUES

1 in. = 25·40 mm
1 mile = 1·609 km
1 gallon = 4·546 litres

(1) *Convert* 7·25 *in. to millimetres.*

$$1 \text{ in.} = 25{\cdot}4 \text{ mm}$$
$$7{\cdot}25 \text{ in.} = 25{\cdot}4 \times 7{\cdot}25 = \underline{184{\cdot}05 \text{ mm}}$$

(2) *Convert* 118 *mm to inches.*

$$25{\cdot}4 \text{ mm} = 1 \text{ in.}$$
$$118 \text{ mm} = \frac{1}{25{\cdot}4} \times 118 = \underline{4{\cdot}646 \text{ in.}}$$

(3) *What is the error in inches in taking* 35·55 *mm as* 1·406 *in.?*

$$25{\cdot}4 \text{ mm} = 1 \text{ in.}$$
$$35{\cdot}55 \text{ mm} = \frac{35{\cdot}55}{25{\cdot}4} = 1{\cdot}400 \text{ in.}$$
$$\text{Error} = 1{\cdot}406 - 1{\cdot}400 = \underline{0{\cdot}006 \text{ in.}}$$

(4) *Convert* 24 *m.p.h. to km/hr.*

$$1 \text{ mile} = 1{\cdot}609 \text{ km}$$
$$24 \text{ miles} = 24 \times 1{\cdot}609 \text{ km} = 38{\cdot}59 \text{ km}$$
$$24 \text{ m.p.h.} = 38{\cdot}59 \text{ km/hr}$$

(5) *Convert 35 miles/gal to km/litre.*

$$\begin{aligned}35 \text{ miles/gal} &= 35 \times 1{\cdot}609 \text{ km/gal}\\ &= \frac{35 \times 1{\cdot}609}{4{\cdot}546} \text{ km/litre}\\ &= \underline{12{\cdot}38 \text{ km/litre}}\end{aligned}$$

20. Algebra: Symbols, Simplifications, Brackets

(1) *If* $a = 2$, $b = 3$, *find the values of:* (*a*) $b + 5$; (*b*) $10 - a - b$; (*c*) $5a$; (*d*) ab; (*e*) b/a; (*f*) $7a - 3b$; (*g*) $\dfrac{2ab - a}{4a + b}$; (*h*) $3(2a + 4b)$.

(*a*) $b + 5 = 3 + 5 = \underline{8}$

(*b*) $10 - a - b = 10 - 2 - 3 = \underline{5}$

(*c*) $5a = 5 \times a = 5 \times 2 = \underline{10}$

(*d*) $ab = a \times b = 2 \times 3 = \underline{6}$

(*e*) $b/a = b \div a = 3 \div 2 = \underline{1\tfrac{1}{2}}$

(*f*) $7a - 3b = (7 \times 2) - (3 \times 3) = 14 - 9 = \underline{5}$

(*g*) $\dfrac{2ab - a}{4a + b} = \dfrac{12 - 2}{8 + 3} = \underline{\dfrac{10}{11}}$

(*h*) $3(2a + 4b) = 3(4 + 12) = 3 \times 16 = \underline{48}$

(2) *If* $x = 5$, $y = 3$, *find the values of:* (*a*) x^2; (*b*) $2y^3$; (*c*) $(7xy)^2$; (*d*) $(2x - y)^3$.

(*a*) $x^2 = x \times x = 5 \times 5 = \underline{25}$

(*b*) $2y^3 = 2 \times y \times y \times y = 2 \times 3 \times 3 \times 3 = \underline{54}$

(*c*) $(7xy)^2 = (7 \times 5 \times 3)^2 = 105^2 = 105 \times 105 = \underline{11{,}025}$

(*d*) $(2x - y)^3 = (10 - 3)^3 = 7^3 = 7 \times 7 \times 7 = \underline{343}$

(3) *Simplify:* (*a*) $4y + 2y + 5y$; (*b*) $\frac{1}{4}h - \frac{1}{8}h + \frac{3}{16}h$; (*c*) $18m - 3 - 6m + 4$.

(*a*) $4y + 2y + 5y = \underline{11y}$

$$(b)\ \tfrac{1}{4}h - \tfrac{1}{8}h + \tfrac{3}{16}h = (\tfrac{1}{4} - \tfrac{1}{8} + \tfrac{3}{16})h = \frac{4-2+3}{16}h = \underline{\tfrac{5}{16}h}$$

$$(c)\ 18m - 3 - 6m + 4 = 18m - 6m - 3 + 4 = \underline{12m + 1}$$

(4) *Simplify:* (*a*) $3x \times 4y$; (*b*) $\frac{1}{8}p \times 4q$; (*c*) $4z \times 5z$; (*d*) $\frac{1}{4}h \times 20h$.

$$(a)\ 3x \times 4y = \underline{12xy}$$

$$(b)\ \tfrac{1}{8}p \times 4q = \tfrac{4}{8}pq = \underline{\tfrac{1}{2}pq}$$

$$(c)\ 4z \times 5z = 4 \times 5z^2 = \underline{20z^2}$$

$$(d)\ \tfrac{1}{4}h \times 20h = \tfrac{20}{4}h^2 = \underline{5h^2}$$

(5) *Simplify:* (*a*) $2(y + 4) + 3(y + 2)$; (*b*) $5(2p - 3) + 4(p + 4)$.

$$(a)\ 2(y + 4) + 3(y + 2) = 2y + 8 + 3y + 6 = \underline{5y + 14}$$

$$(b)\ 5(2p - 3) + 4(p + 4) = 10p - 15 + 4p + 16 = \underline{14p + 1}$$

21. Algebra: Solution of Equations

(1) *Solve the equations:* (*a*) $t + 2 = 6$; (*b*) $h - 7 = 19$; (*c*) $5q = 20$; (*d*) $\frac{1}{6}s = \frac{1}{2}$; (*e*) $\frac{15}{c} = 45$; (*f*) $\frac{3}{w} = \frac{5}{2}$

(*a*) $t + 2 = 6$ $\quad \therefore t = 6 - 2$ $\quad \underline{t = 4}$

(*b*) $h - 7 = 19$ $\quad \therefore h = 19 + 7$ $\quad \underline{h = 26}$

(*c*) $5q = 20$ $\quad \therefore q = \frac{20}{5}$ $\quad \underline{q = 4}$

(*d*) $\frac{1}{6}s = \frac{1}{2}$ $\quad$ Cross multiply $2s = 6$ $\quad \underline{s = 3}$

(*e*) $\frac{15}{c} = 45$ $\quad$ Cross multiply $15 = 45c$ $\quad \underline{c = \frac{15}{45} = \frac{1}{3}}$

(*f*) $\frac{3}{w} = \frac{5}{2}$ $\quad$ Cross multiply $6 = 5w$ $\quad \underline{w = \frac{6}{5} = 1\frac{1}{5}}$

(2) *Solve the equations:* (*a*) $9y + 5 = 38$; (*b*) $6u - 2 = 13$; (*c*) $\frac{1}{5}z - 3 = 2$; (*d*) $7(t - 2) = 21$; (*e*) $4(x + 2) + 3(4x - 5) = 25$; (*f*) $6(5k - 2) = 7(3k + 1) + 17$.

(*a*) $9y + 5 = 38 \quad \therefore\ 9y = 38 - 5 \quad \therefore\ 9y = 33$

$$\underline{y = \frac{33}{9} = 3\tfrac{2}{3}}$$

(*b*) $6u - 2 = 13 \quad \therefore\ 6u = 13 + 2 \quad \therefore\ 6u = 15$

$$\underline{u = \frac{15}{6} = 2\tfrac{1}{2}}$$

(*c*) $\frac{1}{5}z - 3 = 2 \quad \therefore\ \frac{1}{5}z = 2 + 3 \quad \therefore\ \frac{1}{5}z = 5$

$$\underline{z = 5 \times 5 = 25}$$

(*b*) $7(t - 2) = 21 \quad \therefore\ t - 2 = 3 \quad \therefore\ t = 2 + 3$

$$\underline{t = 5}$$

(*e*)
$$\begin{aligned} 4(x + 2) + 3(4x - 5) &= 25 \\ \therefore\ 4x + 8 + 12x - 15 &= 25 \\ 4x + 12x &= 25 - 8 + 15 \\ 16x &= 32 \\ \underline{x = 2} \end{aligned}$$

(*f*)
$$\begin{aligned} 6(5k - 2) &= 7(3k + 1) + 17 \\ \therefore\ 30k - 12 &= 21k + 7 + 17 \\ 30k - 21k &= 7 + 17 + 12 \\ 9k &= 36 \\ \underline{k = 4} \end{aligned}$$

(3) *Solve the equations:* (*a*) $2c^2 = 18$; (*b*) $s^2 + 15 = 40$; (*c*) $3x^2 + 12 = 255$.

(*a*)
$$\begin{aligned} 2c^2 &= 18 \\ \therefore\ c^2 &= 9 \\ c &= \sqrt{9} \\ \underline{c = 3} \end{aligned}$$

(*b*)
$$\begin{aligned} s^2 + 15 &= 40 \\ \therefore\ s^2 &= 40 - 15 \\ s^2 &= 25 \\ s &= \sqrt{25} \\ \underline{s = 5} \end{aligned}$$

$$\begin{aligned}(c)\quad 3x^2 + 12 &= 255\\ 3x^2 &= 255 - 12 = 243\\ x^2 &= 81\\ x &= \sqrt{81}\\ \underline{x} &\underline{= 9}\end{aligned}$$

(4) (*a*) *Solve the equation* $\sqrt{(2)}r + r = 5$; (*b*) *if* $\frac{b}{a-b} = \frac{4}{5}$ *calculate* b *when* $a = 3$.

$$(a)\quad \sqrt{(2)}r + r = 5$$

$$\begin{aligned}\therefore\ 1{\cdot}414r + r &= 5\\ 2{\cdot}414r &= 5\\ \underline{r} &\underline{= \frac{5}{2{\cdot}414} = 2{\cdot}07}\end{aligned}$$

$$(b)\quad \frac{b}{a-b} = \frac{4}{5}$$

When $a = 3$ this becomes

$$\frac{b}{3-b} = \frac{4}{5}$$

Cross multiply:

$$\begin{aligned}5b &= 4(3 - b)\\ 5b &= 12 - 4b\\ 5b + 4b &= 12\\ 9b &= 12\\ \underline{b} &\underline{= \frac{12}{9} = 1\tfrac{1}{3}}\end{aligned}$$

22. Evaluation of Formulae I

(1) *The diameter D of the head of a flat-headed machine screw is given by* $D = 1{\cdot}64d - 0{\cdot}009$, *where* d *is the diameter of the body. Calculate* D *when* $d = 0{\cdot}216$ *in.*

$$\begin{aligned}D &= 1{\cdot}64d - 0{\cdot}009\\ &= 1{\cdot}64 \times 0{\cdot}216 - 0{\cdot}009\\ &= 0{\cdot}3542 - 0{\cdot}0090\\ &= \underline{0{\cdot}3452\text{ in.}}\end{aligned}$$

(2) *The formula* $M = D - 1{\cdot}5155P + 3W$ *is used in the three-wire method of measuring threads. Calculate* M *when* $D = 1\frac{1}{4}$ *in.,* $P = \frac{1}{2}$ *in.,* $W = 0{\cdot}070$ *in.*

$$\begin{aligned} M &= D - 1{\cdot}5155P + 3W \\ &= 1{\cdot}25 - (1{\cdot}5155 \times \tfrac{1}{2}) + (3 \times 0{\cdot}070) \\ &= 1{\cdot}2500 - 0{\cdot}7578 + 0{\cdot}2100 \\ &= \underline{0{\cdot}7022 \text{ in.}} \end{aligned}$$

(3) *When rolling certain types of thread the following formula is used:* $D = n(d - 1{\cdot}25G)$ *where* D *is the outside diameter of the thread roll,* d *is the diameter after completion of the thread,* G *the depth of the thread and* n *the number of threads on the roll. Calculate* D *when* $n = 8$, $d = 0{\cdot}25$, $G = \frac{1}{24}$.

$$\begin{aligned} D &= 8(0{\cdot}25 - 1{\cdot}25 \times \tfrac{1}{24}) \\ &= 8(0{\cdot}2500 - 0{\cdot}0521) \\ &= 8 \times 0{\cdot}1979 = \underline{1{\cdot}5832 \text{ in.}} \end{aligned}$$

(4) *If* F *lb is the tangential force acting on the rim of a flywheel of radius* R *ft and* n *is the rev/min then the horse-power* H *is given by* $H = \dfrac{FRn}{5252}$. *Calculate* H *when* $R = 1\frac{1}{4}$, $F = 60$, $n = 1200$.

$$\begin{aligned} H &= \frac{60 \times 1\frac{1}{4} \times 1200}{3252} \\ &= \frac{90{,}000}{5252} \\ &= \underline{17{\cdot}1} \end{aligned}$$

(5) *The stress on a flywheel rim* S *lb is given by*

$$S = 0{\cdot}00005427WRn^2$$

where W *lb is the weight of the rim,* R *ft is the mean radius of the rim and* n *is the rev/min. Calculate* S *when* $W = 120$, $R = 2$, $n = 340$.

$$\begin{aligned} S &= 0{\cdot}00005427 \times 120 \times 2 \times 340^2 \\ &= \underline{1506 \text{ lb}} \end{aligned}$$

23. Evaluation of Formulae II

(1) *The formula* $Z = h\left(A + \frac{a}{6}\right)$ *is used in calculating the strength of built-up beams. If* $Z = 182$, $h = 28$ *and* $a = 15$, *calculate the value of* A.

$$Z = h\left(A + \frac{a}{6}\right)$$

$$\therefore\ 182 = 28\left(A + \frac{15}{6}\right)$$

$$\frac{182}{28} = A + \frac{15}{6}$$

$$6\tfrac{1}{2} = A + 2\tfrac{1}{2}$$

$$A = 6\tfrac{1}{2} - 2\tfrac{1}{2}$$

$$= \underline{4}$$

(2) *The formula* $p = \frac{45,000}{60v + 20}$ *gives the allowable pressure on slides. Calculate* v *when* $p = 250$.

$$p = \frac{45,000}{60v + 20}$$

$$\therefore\ 250 = \frac{45,000}{60v + 20}$$

$$250(60v + 20) = 45,000$$

$$15,000v + 5000 = 45,000$$

$$15v + 5 = 45$$

$$15v = 45 - 5 = 40$$

$$v = \frac{40}{15}$$

$$= \underline{2\tfrac{2}{3}}$$

(3) *The deflection* D *of a beam of diameter* d *in. whose length is* l *in. and which carries a load of* W *lb at its centre is approximately*

$D = \dfrac{Wl^3}{70 \times 10^6 \times d^4}$. *Calculate the deflection when* $d = \frac{3}{4}$, $l = 10$, $W = 500$.

$$D = \frac{Wl^3}{70 \times 10^6 \times d^4} = \frac{500 \times 10^3}{70 \times 10^6 \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}}$$

$$= \frac{5 \times 4 \times 4 \times 4 \times 4}{70 \times 10 \times 3 \times 3 \times 3 \times 3} = \underline{0{\cdot}023 \text{ in.}}$$

(4) *The formula* $A = \dfrac{0{\cdot}15\sqrt{v} + 9}{P}$ *which occurs in spur gearing gives the connexion between the width of face A in., the diametral pitch P and the pitch-line velocity v in ft/min. Calculate A when* $v = 786$ *and* $P = 5$.

$$A = \frac{0{\cdot}15\sqrt{786} + 9}{5}$$

$$= \frac{(0{\cdot}15 \times 28{\cdot}04) + 9}{5}$$

$$= \frac{4{\cdot}21 + 9}{5} = \frac{13{\cdot}21}{5}$$

$$= \underline{2{\cdot}64 \text{ in.}}$$

(5) *The hardness value of a specimen is given by the formula* $\dfrac{3000}{15{\cdot}71\{10 - \sqrt{(100 - d^2)}\}}$. *Find to the nearest whole number the hardness value when* $d = 8$.

$$\text{Hardness value} = \frac{3000}{15{\cdot}71\{10 - \sqrt{(100 - d^2)}\}}$$

$$= \frac{3000}{15{\cdot}71\{10 - \sqrt{(100 - 64)}\}}$$

$$= \frac{3000}{15{\cdot}71(10 - \sqrt{36})}$$

$$= \frac{3000}{15{\cdot}71 \times 4}$$

$$= \underline{48} \text{ to nearest whole number.}$$

24. Angles

The following are the most important facts about angles. Referring to fig. 13, if the two lines are parallel: (i) the angles marked x are equal and called corresponding angles; (ii) the angles marked y are equal and are called alternate angles.

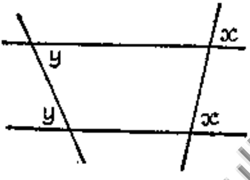


Fig. 13

The sum of the three angles of a triangle is 180°.

A triangle with two equal sides is called isosceles, and the angles opposite the equal sides are equal.

A triangle with all three sides equal is called equilateral and the angles are each 60°.

(1) (*a*) *Express* 53° 27′ *in degrees and decimals of a degree;* (*b*) *Express* 15·22° *in degrees and minutes.*

(*a*) Since $60' = 1°$

then $27' = \frac{27}{60}$ degrees $= 0{\cdot}45°$

$\therefore 53°\,27' = \underline{53{\cdot}45°}$

(*b*) Since $1° = 60'$

then $0{\cdot}22° = 60 \times 0{\cdot}22 = 13{\cdot}2'$

$\therefore 15{\cdot}22° = \underline{15°\,13'\text{ to the nearest minute.}}$

(2) *If two angles of a triangle are* 63° 36′ *and* 44° 48′, *calculate the remaining angle.*

Since the sum of the three angles of a triangle is 180°, the third angle $= 180° - (63°\,36' + 44°\,48')$

$= 180° - 108°\,24'$

$= \underline{71°\,36'}$

(3) *In fig.* 14 *AD bisects* $\angle BAC$. *Calculate* θ.

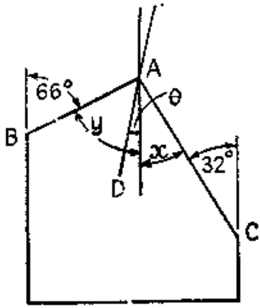


Fig. 14

$$\angle x = 32^\circ \qquad \angle y = 66^\circ$$

In both cases, these are alternate angles

$$\therefore \ \angle BAC = x + y = 66^\circ + 32^\circ = 98^\circ$$

Since AD bisects this angle

$$\angle CAD = x + \theta = \tfrac{1}{2}(98^\circ) = 49^\circ$$

$$\therefore \theta = 49^\circ - x$$

$$= 49^\circ - 32^\circ$$

$$= \underline{17^\circ}$$

(4) *The triangle ABC shown in fig.* 15 *has a right angle at A and the sides BD, DC, and AD are all equal. Find the angles of the triangle ABD and ADC.*

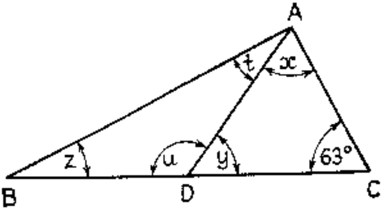


Fig. 15

Since $DA = DC$

$$\therefore\ x = 63^\circ \text{ (isosceles triangle)}$$
$$y = 180^\circ - (63^\circ + 63^\circ) = 180^\circ - 126^\circ = 54^\circ$$
$$u = 180^\circ - y = 180^\circ - 54^\circ = 126^\circ$$

Therefore the sum of the other two angles

$$t + z = 180^\circ - 126^\circ = 54^\circ$$

But $t = z$ (isosceles triangle)

therefore $t = \frac{1}{2} \times 54^\circ = \underline{27^\circ}$ and $z = \underline{27^\circ}$

(5) *Calculate the angle marked θ shown in fig. 16. BG and CF are parallel to AO.*

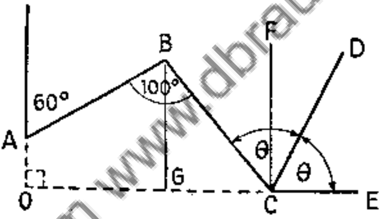


Fig. 16

$$\angle ABG = \angle BAO = 60^\circ \text{ (alternate angles)}$$
$$\therefore\ \angle GBC = 100^\circ - 60^\circ = 40^\circ$$
But $$\angle BCF = \angle GBC = 40^\circ \text{ (alternate angles)}$$
$$\therefore\ BCE = 40^\circ + 90^\circ = 130^\circ$$
$$\therefore\ \theta = \tfrac{1}{2} \times 130^\circ$$
$$= \underline{65^\circ}$$

25. Pythagoras' Theorem

The Theorem of Pythagoras states that in a right-angled triangle, the square on the hypotenuse (the side opposite the right angle) is equal to the sum of the squares on the other two sides.

Referring to fig. 17 if C is a right angle, then $c^2 = a^2 + b^2$

(1) *Referring to fig. 17: (a) if $a = 4$ in. and $b = 6$ in., calculate c;*

(b) if $c = 14{\cdot}31$ *in. and* $a = 11{\cdot}58$ *in., calculate* b*; (c) if* $c = 7\frac{5}{8}$ *in. and* $b = 3\frac{1}{4}$ *in. calculate* a.

(a)
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 4^2 + 6^2 \\ &= 16 + 36 = 52 \\ c &= \sqrt{52} = \underline{7{\cdot}21 \text{ in.}} \end{aligned}$$

(b)
$$\begin{aligned} c^2 &= a^2 + b^2 \\ \therefore\ b^2 &= c^2 - a^2 \\ &= 14{\cdot}31^2 - 11{\cdot}58^2 \\ &= 204{\cdot}8 - 134{\cdot}1 \\ &= 70{\cdot}7 \\ b &= \sqrt{70{\cdot}7} = \underline{8{\cdot}41 \text{ in.}} \end{aligned}$$

(c)
$$\begin{aligned} c^2 &= a^2 + b^2 \\ \therefore\ a^2 &= c^2 - b^2 \\ &= 7{\cdot}625^2 - 3{\cdot}25^2 \\ &= 58{\cdot}14 - 10{\cdot}56 \\ &= 47{\cdot}58 \\ a &= \sqrt{47{\cdot}58} = \underline{6{\cdot}90 \text{ in.}} \end{aligned}$$

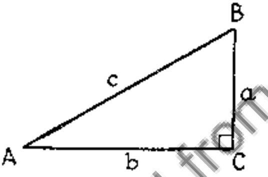


Fig. 17

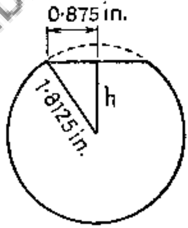


Fig. 18

(2) *A flat* $1\frac{3}{4}$ *in. wide is to be machined on a bar* $3\frac{5}{8}$ *in. diameter. Calculate the maximum depth of cut (fig. 18).*

Radius of bar $= \frac{1}{2}(3{\cdot}625) = 1{\cdot}8125$ in.

Half length of flat $= \frac{1}{2}(1{\cdot}75) = 0{\cdot}875$ in.

By Pythagoras' Theorem
$$\begin{aligned} h^2 &= 1{\cdot}8125^2 - 0{\cdot}875^2 \\ &= 3{\cdot}285 - 0{\cdot}766 = 2{\cdot}519 \\ h &= \sqrt{2{\cdot}519} = 1{\cdot}5871 \text{ in.} \end{aligned}$$

$$\begin{aligned} \text{Depth of cut} &= 1{\cdot}8125 - 1{\cdot}5871 \\ &= 0{\cdot}2254 = \underline{0{\cdot}225} \text{ to the nearest thousandth of an inch} \end{aligned}$$

(3) *A triangle ABC has sides AB and AC of lengths* 5·64 *in. and* 3·22 *in. respectively and the length of the perpendicular from C to AB is* 2·65 *in. Calculate the length of BC (fig.* 19).

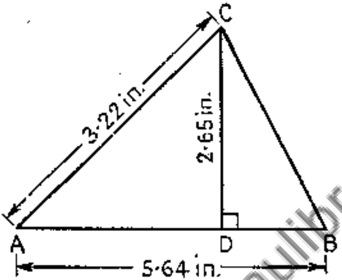


Fig. 19

$$AD^2 = 3{\cdot}22^2 - 2{\cdot}65^2 = 10{\cdot}37 - 7{\cdot}02 = 3{\cdot}35$$
$$AD = \sqrt{3{\cdot}35} = 1{\cdot}83 \text{ in.}$$
$$DB = 5{\cdot}64 - 1{\cdot}83 = 3{\cdot}81 \text{ in.}$$
$$BC^2 = 3{\cdot}81^2 + 2{\cdot}65^2 = 14{\cdot}52 + 7{\cdot}02 = 21{\cdot}54$$
$$BC = \sqrt{21{\cdot}54} = \underline{4{\cdot}64 \text{ in.}}$$

(4) *A keyway is to be cut into a shaft as shown in fig.* 20. *Calculate the depth of the keyway.*

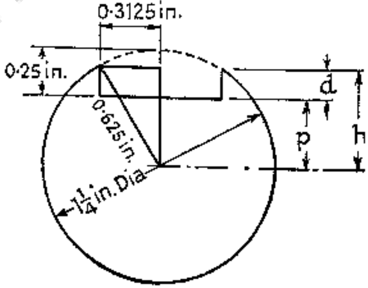


Fig. 20

Radius of circle = $\frac{1}{2}(1{\cdot}25) = 0{\cdot}625$ in.

Half width of keyway = 0·3125 in.

By Pythagoras' Theorem

$$\begin{aligned} h^2 &= 0{\cdot}625^2 - 0{\cdot}3125^2 \\ &= 0{\cdot}3906 - 0{\cdot}0977 \\ &= 0{\cdot}2929 \\ h &= \sqrt{0{\cdot}2929} = 0{\cdot}5412 \text{ in.} \\ p &= 0{\cdot}625 - 0{\cdot}25 = 0{\cdot}375 \text{ in.} \\ d &= h - p = 0{\cdot}5412 - 0{\cdot}375 \\ &= 0{\cdot}1662 \\ &= \underline{0{\cdot}166 \text{ in.}} \text{ correct to three decimal places.} \end{aligned}$$

26. Trigonometry: The Tangent

TRIGONOMETRICAL RATIOS

Referring to fig. 21, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

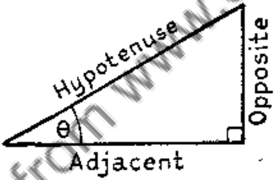


Fig. 21

SPECIMEN LINES FROM TANGENT TABLES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0° ·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36

(1) *Find tan 46° 24′ and tan 46° 47′.*

$$\tan 46^\circ\, 24' = \underline{1{\cdot}0501}$$
$$\begin{aligned}\tan 46^\circ\, 47' &= \tan 46^\circ\, 42' \text{ together with difference for } 5' \\ &= 1{\cdot}0612 + 0{\cdot}0031 = \underline{1{\cdot}0643}\end{aligned}$$

(2) *Find the angle whose tangent is* 1·0593.

$$\begin{aligned}\tan 46^\circ\, 36' &= 1{\cdot}0575 \\ 3' &= 0{\cdot}0018 \\ \tan 46^\circ\, 39' &= \underline{1{\cdot}0593}\end{aligned}$$

(3) *Find the length of the side a in fig.* 22.

$$\frac{a}{7{\cdot}6} = \tan 25^\circ$$

$$\begin{aligned}a &= 7{\cdot}6 \times \tan 25^\circ \\ &= 7{\cdot}6 \times 0{\cdot}4663 \\ &= \underline{3{\cdot}54 \text{ in.}}\end{aligned}$$

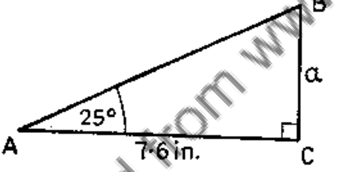


Fig. 22

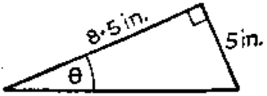


Fig. 23

(4) *Find the angle θ in fig.* 23.

$$\tan \theta = \frac{5}{8{\cdot}5} = 0{\cdot}5882$$

$$\underline{\theta = 30^\circ\, 28'}$$

(5) *Calculate the distance d in fig.* 24.

We work with right-angled triangles and obtain d by

$$d = CD - BC$$

that is, from the two right-angled triangles ACD and ACB.

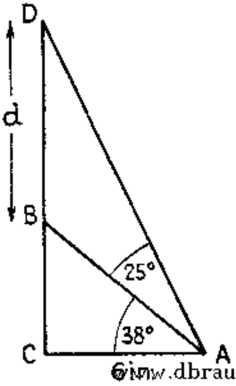


Fig. 24

From $\triangle DCA$ $\dfrac{DC}{6} = \tan 60°$ $DC = 6 \times \tan 60° = 6 \times 1{\cdot}732$

$= 10{\cdot}392$ in.

From $\triangle BCA$ $\dfrac{BC}{6} = \tan 38°$ $BC = 6 \tan 38° = 6 \times 0{\cdot}7813$

$= 4{\cdot}688$ in.

$d = 10{\cdot}392 - 4{\cdot}688 = \underline{5{\cdot}704 \text{ in.}}$

27. Trigonometry: The Sine

In fig. 21, $\sin \theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

SPECIMEN LINES FROM SINE TABLES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
21	·3584	2600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13

(1) *Find sin* 23° 12′ *and sin* 23° 34′.

$$\sin 23^\circ 12' = 0{\cdot}3939$$
$$\sin 23^\circ 34' = \sin (23^\circ 30' + 4')$$
$$= 0{\cdot}3987 + 0{\cdot}0011 = \underline{0{\cdot}3998}$$

(2) *Find the angle whose sine is* 0·4046.

The largest value less than 0·4046 in the table is

$$\sin 23^\circ 48' = 0{\cdot}4035$$

which thus needs an addition of 0·0011 from the difference column.

The angle is, therefore, $23^\circ 48' + 4' = \underline{23^\circ 52'}$.

(3) *In fig.* 25 *find the length x.*

$$\frac{x}{7} = \sin 41^\circ$$

$$x = 7 \sin 41^\circ$$
$$= 7 \times 0{\cdot}6561$$
$$= \underline{4{\cdot}593 \text{ ft}}$$

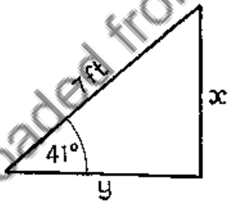


Fig. 25

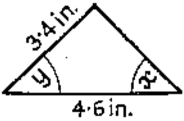


Fig. 26

(4) *Calculate the angle x in fig.* 26.

$$\sin x = \frac{3{\cdot}4}{4{\cdot}6} = 0{\cdot}7391$$

$$x = \underline{47^\circ 39'}$$

(5) *In fig.* 19 *calculate the angle A and use the value of this angle to find the length of AD.*

$$\sin A = \frac{CD}{AC} = \frac{2{\cdot}65}{3{\cdot}22} = 0{\cdot}8230$$

$$A = 55^\circ\ 23'$$

$$\tan A = \frac{CD}{AD}$$

$$\tan 55^\circ\ 23' = \frac{2{\cdot}65}{AD}$$

$$1{\cdot}450 \times AD = 2{\cdot}65$$

$$AD = \frac{2{\cdot}65}{1{\cdot}45} = \underline{1{\cdot}83 \text{ in.}}$$

28. Trigonometry: The Cosine

In fig. 21, $\cos \theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

SPECIMEN LINES FROM COSINE TABLES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1′	2′	3′	4′	5′
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14

(1) *Find cos* 72° 6′ *and cos* 72° 45′.

$$\cos 72^\circ\ 6' = \underline{0{\cdot}3074}$$

$$\cos 72^\circ\ 45' = 0{\cdot}2974 - 0{\cdot}0008 = \underline{0{\cdot}2966}$$

It should be noted that since the cosine gets less as the angle increases the differences must be subtracted.

(2) *Find the angle whose cosine is* 0·2999.

The smallest value greater than 0·2999 is 0·3007, which thus requires a subtraction of 0·0008 which is found in the difference column under 3′.

Hence, the angle is 72° 30′ + 3′ = $\underline{72° 33′}$.

(3) *In fig.* 25, *find the length y.*

$$\frac{y}{7} = \cos 41°$$

$$y = 7 \times 0{\cdot}7547$$

$$= \underline{5{\cdot}283 \text{ ft}}$$

(4) *In fig.* 19, *calculate the angle ACD.*

$$\cos ACD = \frac{2{\cdot}65}{3{\cdot}22} = 0{\cdot}8230$$

$$ACD = \underline{34° 37′}$$

(5) *Calculate the length of AB in fig.* 24.

$$\frac{AC}{AB} = \cos 38°$$

$$\frac{6}{AB} = \cos 38°$$

$$6 = AB \cos 38° \quad AB = \frac{6}{\cos 38°} = \frac{6}{0{\cdot}7880}$$

$$AB = \underline{7{\cdot}62 \text{ in.}}$$

29–30. Trigonometrical Problems

(1) *The difference in height between the centres of the discs of a* 10-*in. sine bar is* 2·345 *in. What angle is indicated?*

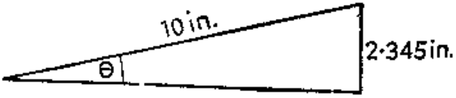


Fig. 27

If, in fig. 27, θ is the angle required, then

$$\sin\theta = \frac{2{\cdot}345}{10} = 0{\cdot}2345$$

$$\theta = \underline{13^\circ\ 34'}$$

(2) *Ten holes are equally spaced round a circle of diameter* 15 *in. Calculate the chordal distance between the centres of two adjacent holes.*

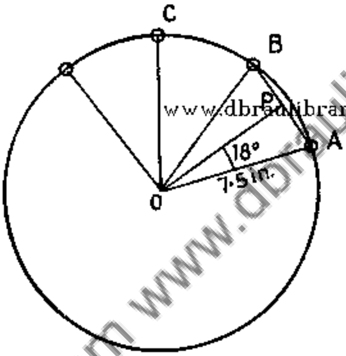


Fig. 28

In fig. 28, the angle subtended at the centre by a pair of adjacent holes $= \dfrac{360^\circ}{10} = 36^\circ$.

If OP is drawn perpendicular to AB, then since $AO = BO$, P bisects AB and OP bisects $\angle AOB$

$$\therefore\ \frac{PB}{OB} = \sin 18^\circ \quad \text{and} \quad \frac{PB}{7{\cdot}5} = 0{\cdot}3090$$

$$\begin{aligned} PB &= 7{\cdot}5 \times 0{\cdot}3090 \\ &= 2{\cdot}318 \end{aligned}$$

$$AB = 2 \times PB = 2 \times 2{\cdot}218 = \underline{4{\cdot}636 \text{ in.}}$$

(3) *Calculate the distance d in the template shown in fig.* 29. (*Construction lines for the solution are dotted.*)

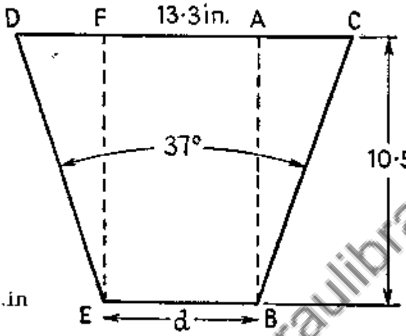


Fig. 29

Draw right-angled triangles ACB and FDE

Then $\angle ABC = \angle FED = 18\tfrac{1}{2}° = 18° \; 30'$

and $DF = AC$

In $\triangle ACB$

$$\frac{AC}{AB} = \tan 18° \; 30'$$

$$\frac{AC}{10{\cdot}5} = 0{\cdot}3346$$

$$\therefore \; AC = 10{\cdot}5 \times 0{\cdot}3346$$
$$= 3{\cdot}513 \text{ in.}$$

Hence

$$d = 13{\cdot}3 - DF - AC$$
$$= 13{\cdot}3 - 2 \times 3{\cdot}513$$
$$= 13{\cdot}3 - 7{\cdot}026$$
$$= \underline{6{\cdot}274 \text{ in.}}$$

(4) *A* 1-*in. test plug is placed in the angular groove shown in fig.* 30. *Calculate the distance d.*

Join OC. If P and Q are the points of contact of the circle and

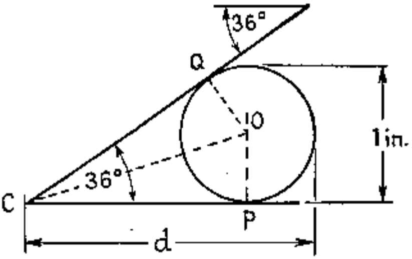


Fig. 30

the tangents, then the angles at P and Q are right angles and by symmetry OC bisects the angle of 36°.

$$\therefore \angle OCP = 18°$$

$\therefore$ from the triangle OPC $\quad \dfrac{OP}{CP} = \tan 18°$

and $\quad OP = 0{\cdot}5$ in.

$$\therefore 0{\cdot}5 = CP \tan 18°$$

and $\quad CP = \dfrac{0{\cdot}5}{\tan 18°} = 1{\cdot}54$ in.

$$d = CP + 0{\cdot}5 = 1{\cdot}54 + 0{\cdot}50 = \underline{2{\cdot}04 \text{ in.}}$$

(5) *Three holes are bored in a plate with centres A, B and C where AB = 16 in., BC = 12 in. and ABC = 40°. Calculate the length of AC (fig. 31).*

Draw DC perpendicular to AB
In $\triangle BCD$

$$\frac{CD}{12} = \sin 40°$$

$$\therefore CD = 12 \times 0{\cdot}6428$$
$$= 7{\cdot}714 \text{ in.}$$

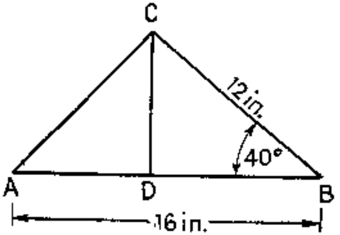


Fig. 31

Also in $\triangle BCD$ $\qquad \dfrac{BD}{12} = \cos 40°$

$$\begin{aligned} \therefore\ BD &= 12 \times 0{\cdot}7660 \\ &= 9{\cdot}192 \text{ in.} \\ \therefore\ AD &= 16 - 9{\cdot}192 \\ &= 6{\cdot}808 \text{ in.} \end{aligned}$$

In $\triangle ADC$ $\qquad AD = 6{\cdot}808 \qquad CD = 7{\cdot}714$

$\therefore$ by Pythagoras' Theorem

$$\begin{aligned} AC^2 &= 6{\cdot}808^2 + 7{\cdot}714^2 \\ &= 46{\cdot}35 + 59{\cdot}50 \\ &= 105{\cdot}85 \\ AC &= \sqrt{105{\cdot}85} = \underline{10{\cdot}29 \text{ in.}} \end{aligned}$$

(6) *For the template shown in fig.* 32, *calculate*: (i) *the length of* BC *if* $\angle CAB = 90$; (ii) *the angles B and C of the triangle*; (iii) *the perimeter of the template.*

By Pythagoras' Theorem

$$\begin{aligned} BC^2 &= CA^2 + AB^2 \\ &= 5^2 + 12^2 = 169 \qquad BC = \sqrt{169} = 13 \text{ in.} \end{aligned}$$

To find angle B

$$\begin{aligned} \tan B &= \tfrac{5}{12} = 0{\cdot}4167 \\ B &= 22°\ 37' \\ \text{angle } C &= 90° - 22°\ 37' = 67°\ 23' \end{aligned}$$

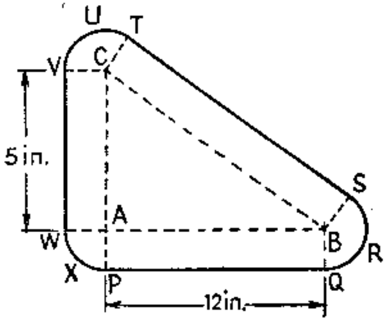


Fig. 32

Angle subtended by arc *QRS* at centre

$$= 360° - 90° - 22° 37' - 90°$$
$$= 180° - 22° 37'$$

Angle subtended by arc *TUV* at centre

$$= 360° - 90° - 67° 23' - 90°$$
$$= 180° - 67° 23'$$

Angle subtended by arc *WXP* at centre = 90°
Total angle subtended by three arcs

$$= 180° - 22° 37' + 180° - 67° 23' + 90°.$$
$$= 360°$$

Total perimeter of three arcs = 2π = 6·28 in.
Total perimeter of template = 6·28 + 5 + 12 + 13 = 36·28 in.

31. Taper Turning

(1) *Calculate the tailstock set-over to turn a taper of:* (*a*) 1 *in* 10 *on a job* 24-*in. long;* (*b*) 16° *on a job* 20-*in. long.*

If the tailstock of the lathe is set over a distance *d* then 2 × *d* in. more will be removed at the tailstock end than at the other end.

(*a*) For a taper of 1 in 10 on a job 24-in. long, the difference in the diameters of the two ends $= \frac{24}{10} = 2{\cdot}4$ in.

$$\therefore\ 2d = 2{\cdot}4$$

$$\therefore\ \text{tailstock set-over, } d = \underline{1{\cdot}2 \text{ in.}}$$

(*b*) Difference between the diameters of the two ends

$$= 2 \times 20 \tan 8°$$
$$= 40 \times 0{\cdot}1405 = 5{\cdot}62 \text{ in.}$$

$$\therefore\ 2d = 5{\cdot}62$$

$$\therefore\ \text{tailstock set-over, } d = \underline{2{\cdot}81 \text{ in.}}$$

(2) *Find the setting of the compound slide to turn a taper of* 1 *in* 8 *in the job shown in fig.* 33.

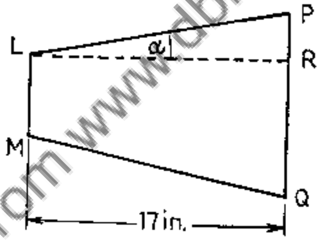


Fig. 33

Since the job is 17-in. long the longer diameter is $\frac{17}{8} = 2{\cdot}125$ in. greater than the smaller diameter

$$PR = \frac{2{\cdot}125}{2} = 1{\cdot}0625$$

If α is the angle required, $\tan \alpha = \frac{1{\cdot}0625}{17}$

$$= 0{\cdot}0625$$

$$\underline{\alpha = 3° \ 34'}$$

(3) (*a*) *If the taper shown in fig.* 34 *is turned by offsetting the*

tailstock, calculate the offset; (b) If the taper is turned by using the compound slide, calculate the setting.

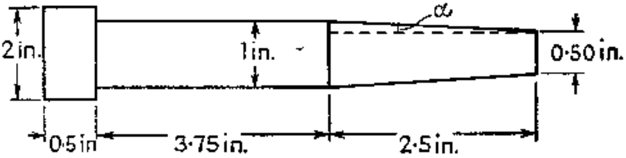


Fig. 34

(*a*) Total taper = 1·00 − 0·50 = 0·50 in.
If the job were 2·50-in. long set-over would be 0·25 in.

Since the job is 6·75-in. long the set-over $= \dfrac{0{\cdot}25 \times 6{\cdot}75}{2{\cdot}50}$

$= \underline{0{\cdot}675 \text{ in.}}$

(*b*) If the compound slide is used and α is the setting angle

$$\tan \alpha = \frac{0{\cdot}25}{2{\cdot}50} = 0{\cdot}10$$

$$\alpha = \underline{5^\circ\, 42'}$$

32. The Circle: Trigonometrical Problems

(1) *A 60° vee-block is 1·250-in. wide at the top. If a 0·750 in. diameter plug is placed in the groove, find the height of the top of the plug above the top of the vee.*

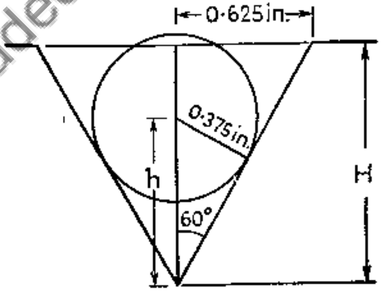


Fig. 35

If H is the height of the triangle (fig. 35)

$$\frac{0{\cdot}625}{H} = \tan 30^\circ$$

$$H = \frac{0{\cdot}625}{\tan 30^\circ} = 1{\cdot}083 \text{ in.}$$

Distance of the centre of the plug from the bottom of the triangle is given by h where $\frac{0{\cdot}375}{h} = \sin 30^\circ$

$$h = \frac{0{\cdot}375}{\sin 30^\circ} = 0{\cdot}750 \text{ in.}$$

Distance of top of plug from bottom of triangle

$$\begin{aligned} &= 0{\cdot}750 + 0{\cdot}375 \\ &= 1{\cdot}125 \text{ in.} \end{aligned}$$

Distance of plug from top of vee $= 1{\cdot}125 - 1{\cdot}083 = \underline{0{\cdot}042 \text{ in.}}$

(2) *Calculate the diameter of a plug which, when placed in a* 150° *vee-block* 1·500-*in. wide at the top, will have its top surface just level with the top of the block.*

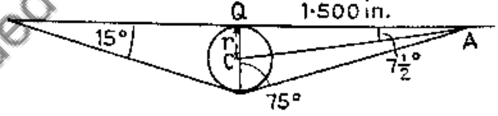


Fig. 36

Join CA (fig. 36). This line then bisects $\angle A$ which is 15°.

In $\triangle CAQ$ $\quad \frac{r}{1{\cdot}500} = \tan 7\tfrac{1}{2}^\circ$

$$r = 1{\cdot}500 \times 0{\cdot}1317 = 0{\cdot}197$$

$$\therefore \text{ Diameter of plug} = \underline{0{\cdot}394 \text{ in.}}$$

(3) *Find the diameter of the largest hole that can be drilled in a quadrant of a circle of radius* 1 *in.*

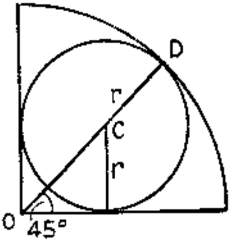


Fig. 37

In fig. 37, since

$$OD = OC + CD$$

$$OC = \frac{r}{\sin 45^\circ} = \frac{r}{0{\cdot}7071} = 1{\cdot}414r$$

$$\therefore\ 1 = 1{\cdot}414r + r$$
$$\therefore\ 1 = 2{\cdot}414r$$

$$r = \frac{1}{2{\cdot}414} = 0.414 \text{ in.}$$

Diameter of the hole = 0·828 in.

(4) *A* 120° *vee-block* 1·500-*in. wide at the top is rounded off at the bottom by a radius of* 0·400 *in. Calculate the maximum depth of the block* (fig. 38).

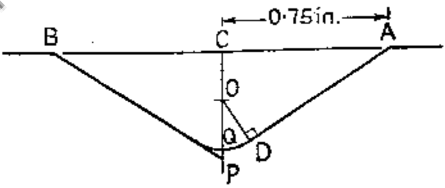


Fig. 38

In fig. 38, C is mid point of BA.

In $\triangle ACP$ $\quad \frac{0{\cdot}75}{PC} = \tan 60° = 1{\cdot}732$

$$\therefore\ PC = \frac{0{\cdot}75}{1{\cdot}732} = 0{\cdot}433$$

In $\triangle ODP$ $\quad \frac{OD}{OP} = \sin 60° = 0{\cdot}866$

$$OP = \frac{OD}{0{\cdot}866} = \frac{0{\cdot}4}{0{\cdot}866} = 0{\cdot}462 \text{ in.}$$

$$\therefore\ QP = OP - OQ$$
$$= 0{\cdot}462 - 0{\cdot}400 = 0{\cdot}062 \text{ in.}$$

$$\therefore\ CQ = CP - QP = 0{\cdot}433 - 0{\cdot}062$$
$$= 0{\cdot}371 \text{ in.}$$

Depth of block = $\underline{0{\cdot}371 \text{ in.}}$

33. The Circle: Applications of Pythagoras' Theorem

(1) *Calculate dimension d in fig.* 39.

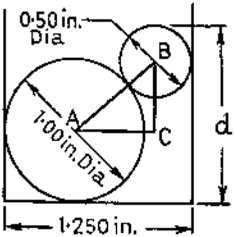


Fig. 39

Complete the triangle ABC, then

$$AC = 1{\cdot}25 - 0{\cdot}50 - 0{\cdot}25 = 0{\cdot}50 \text{ in.}$$
$$AB = 0{\cdot}50 + 0{\cdot}25 = 0{\cdot}75 \text{ in.}$$

By Pythagoras' Theorem

$$\begin{aligned} AB^2 &= 0{\cdot}75^2 - 0{\cdot}50^2 \\ &= 0{\cdot}5625 - 0{\cdot}2500 \\ &= 0{\cdot}3125 \\ AB &= 0{\cdot}559 \text{ in.} \\ d &= 0{\cdot}500 + 0{\cdot}559 + 0{\cdot}250 \\ &= \underline{1{\cdot}309 \text{ in.}} \end{aligned}$$

(2) *Calculate the distance d in fig.* 40.

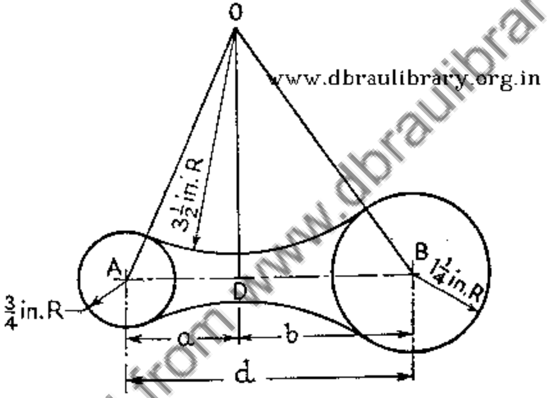


Fig. 40

Complete the triangle OAB and draw OD perpendicular to AB.

Then,

$$\begin{aligned} OA &= 3\tfrac{1}{2} + \tfrac{3}{4} = 4\tfrac{1}{4} \text{ in.} \\ OB &= 3\tfrac{1}{2} + 1\tfrac{1}{4} = 4\tfrac{3}{4} \text{ in.} \\ OD &= 3\tfrac{1}{2} + \tfrac{1}{2} = 4 \text{ in.} \end{aligned}$$

By Pythagoras' Theorem

$$\begin{aligned} a^2 &= 4\tfrac{1}{4}^2 - 4^2 = 18{\cdot}06 - 16 = 2{\cdot}06 \\ a &= \sqrt{2{\cdot}06} = 1{\cdot}44 \\ b^2 &= 4\tfrac{3}{4}^2 - 4^2 = 22{\cdot}56 - 16 = 6{\cdot}56 \\ b &= \sqrt{6{\cdot}56} = 2{\cdot}56 \\ d &= a + b = 1{\cdot}44 + 2{\cdot}56 = \underline{4{\cdot}00 \text{ in.}} \end{aligned}$$

(3) *Find the radius r of the plugs shown in fig.* 41.

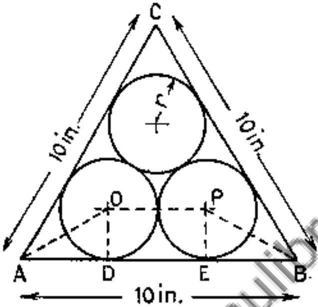


Fig. 41

In the triangle *AOD*

$$\frac{r}{AD} = \tan 30^\circ$$

$$AD = \frac{r}{\tan 30^\circ} = \frac{r}{0{\cdot}5774} = 1{\cdot}732r$$

Since $AD + DE + EB = 10$

then $1{\cdot}732r + 2r + 1{\cdot}732r = 10$

$$\therefore\ 5{\cdot}464r = 10$$

$$r = 10/5{\cdot}464$$

$$= \underline{1{\cdot}83 \text{ in.}}$$

34 The Circle: Length of Belts

(1) *Calculate the length of the belt passing round two pulleys each 15-in. diameter if the distance between the centre of the pulleys is 4 ft 6 in.*

Since the pulleys have equal diameters, the straight sections of the belt are both equal to 4 ft 6 in., and the curved sections will be semicircles (fig. 42).

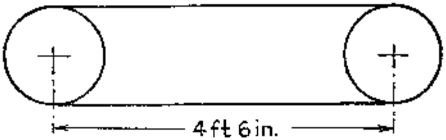


Fig. 42

Length of straight sections $= 2 \times 4$ ft 6 in.

$= 9$ ft

Length of curved section $= 2 \times \left(\dfrac{\pi d}{2}\right)$

$= \pi d$

$= \pi \times 15$

$= 47{\cdot}12$ in.

$= 3$ ft $11{\cdot}4$ in.

Total length $= 9$ ft $+ 3$ ft $11{\cdot}4$ in.

$= \underline{12 \text{ ft } 11{\cdot}4 \text{ in.}}$

(2) *Two pulleys, 2 ft and 1 ft 6 in. diameter, are connected by a crossed belt, the straight sections of which are at 90° to each other. Calculate the length of the belt (fig. 43).*

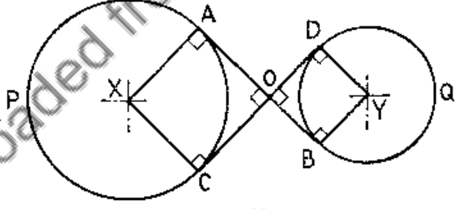


Fig. 43

Since $\angle AOC$ is a right angle and $\angle XAO$ and $\angle XCO$ are each a right angle (angle between tangent and radius = 1 right angle), $XAOC$ is a square and AO and OC are both equal to radius AX. Length of straight sections

$$= 2(AO + OB) = 2(1 + 0{\cdot}75) = 3{\cdot}5 \text{ ft}$$

Since AXC is a right angle, length of arc APC is three-quarters of the circumference of circle.

Length $APC = \frac{3}{4}\pi d = \frac{3}{4} \times \pi \times 1 = 2{\cdot}36$ ft
Similarly, length of $DQB = \frac{3}{4}\pi \times 0{\cdot}75 = 1{\cdot}77$ ft
Total length of belt $= 3{\cdot}5 + 2{\cdot}36 + 1{\cdot}77 = \underline{7{\cdot}63 \text{ ft}}$

(3) *Two pulleys 1-ft and 3-ft diameter respectively are connected by an open belt. If the distance between the centres of the pulleys is 5 ft, calculate the length of belting* (*fig.* 44).

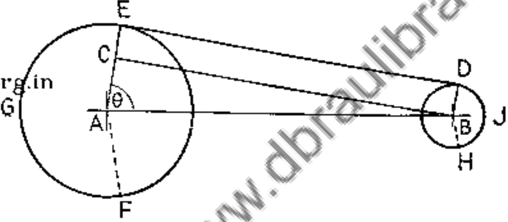


Fig. 44

AC = difference in radii = 1 ft.

By Pythagoras' Theorem

$$AB^2 = AC^2 + CB^2$$
$$5^2 = 1^2 + CB^2$$
$$CB^2 = 24$$
$$CB = \sqrt{24} = 4{\cdot}90 \text{ ft}$$
$$\cos\theta = \tfrac{1}{5} = 0{\cdot}2000$$
$$\therefore \theta = 78^\circ 28'$$

Sector angle EGF

$$= 360^\circ - (2 \times 78^\circ 28') = 360^\circ - 156^\circ 56' = 203^\circ 4'$$

Length of arc EGF

$$= \pi d \times \frac{203^\circ 4'}{360} = \pi \times 3 \times \frac{203{\cdot}1}{360} = 5{\cdot}32 \text{ ft}$$

Sector angle DJH

$$= 156^\circ 56'$$

Length of arc DJH

$$= \pi \times 1 \times \frac{156{\cdot}9}{360} = 1{\cdot}37 \text{ ft}$$

Total length

$$= (2 \times 4{\cdot}90) + 5{\cdot}32 + 1{\cdot}37 = \underline{16{\cdot}49 \text{ ft}}$$

(4) *If, in Example* (3), *a crossed belt is used, calculate the length of belting required* (*see fig.* 45).

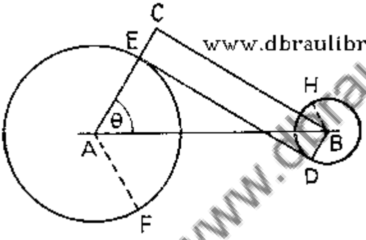


Fig. 45

$$AC = \text{sum of radii} = \tfrac{1}{2}(1 + 3) = 2 \text{ ft}$$

By Pythagoras' Theorem

$$AB^2 = AC^2 + CB^2$$
$$5^2 = 2^2 + CB^2$$
$$CB^2 = 21 \qquad CB = \sqrt{21} = 4{\cdot}58 \text{ ft}$$
$$\cos\theta = \tfrac{2}{5} = 0{\cdot}4000$$
$$\therefore\ \theta = 66^\circ\ 25'$$

Sector angle EGF

$$= 360^\circ - (2 \times 66^\circ\ 25') = 360^\circ - 132^\circ\ 50' = 227^\circ\ 10'$$

Length of arc EGF

$$= \pi d \times \frac{227^\circ\ 10'}{360^\circ} = 3\pi \times \frac{227{\cdot}2}{360} = 5{\cdot}95 \text{ ft}$$

Length of *HJD*

$$= \pi d \times \frac{227^\circ\ 10'}{360^\circ} = \pi\frac{227{\cdot}2}{360} = 1{\cdot}98 \text{ ft}$$

Total length

$$= (2 \times 4{\cdot}58) + 5{\cdot}94 + 1{\cdot}98 = \underline{17{\cdot}08 \text{ ft}}$$

35. Graphs from Tables of Values

(1) *The speed of a belt over a pulley rotating at a constant speed depends on the diameter of the pulley, the connexion being given by the following table:*

Diameter (in.)	2	4	6	8	10	12
Speed (ft/min)	73	147	220	293	367	440

Plot a graph and from it find the speed of the belt if a $6\frac{1}{4}$ *in. diameter pulley is used.*

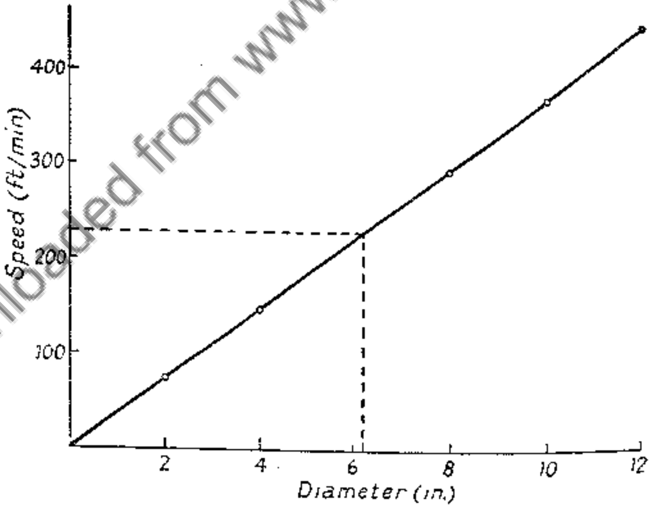


Fig. 46

For a $6\frac{1}{4}$-in. diameter pulley, speed = <u>229 ft/min</u>

(2) *The weight per foot length of round iron bar for diameters between* 1 *in. and* 5 *in. are given in the following table. Plot a graph and from it find the weight of the casting shown in fig.* 47(*b*).

Dia (in.)	1	2	3	4	5
Wt (lb)	2·9	11·7	25·8	46·8	73·0

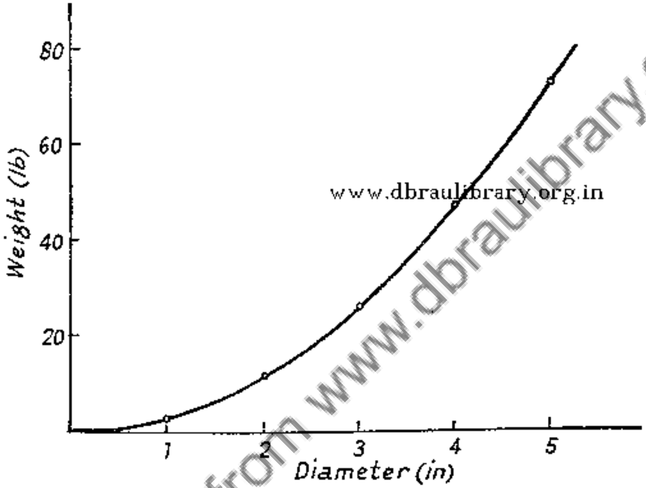


Fig. 47(*a*)

From graph (fig. 47(*a*))

Weight per foot length of $4\frac{1}{2}$-in. dia bar = 59 lb
Weight of 1 in. = 4·92 lb

Weight per foot length of $3\frac{1}{2}$-in. dia bar = 35 lb
Weight of 3-in. length = 8·75 lb

Weight per foot length of $2\frac{1}{2}$-in. dia bar = 18 lb
Weight of 2-in. length = 3·00 lb

Weight of 6-in. length of 1-in. dia bar = 1·45 lb

Total Weight = 4·92 + 8·75 + 3·00 − 1·45

$= \underline{15{\cdot}22 \text{ lb}}$

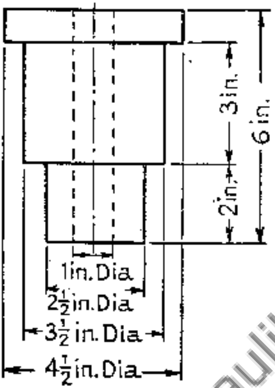


Fig. 47(*b*)

36. Co-ordinate Dimensions

(1) *On a sheet of graph paper plot the points whose co-ordinates are: A,* (3,5) *B,* (−2,−1) *C,* (−4,2) *D,* (3,−1)

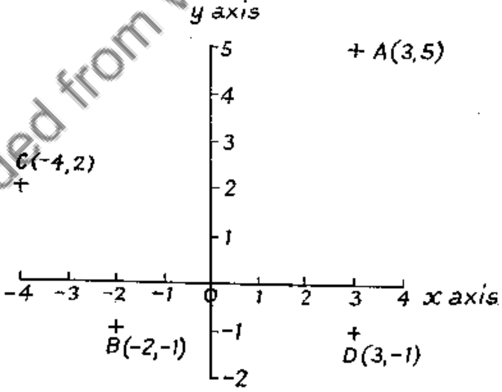


Fig. 48

Two axes are drawn to meet at *O*. The horizontal axis is called the *x* axis and the vertical axis is called the *y* axis. On the *x* axis units to the right are + and units to the left are −. On the y axis units upwards are + and units downwards are −. The distances which specify the position of a point are called its co-ordinates and they are written in the form $(-2,-1)$ the x co-ordinate being first and the y co-ordinate second. These co-ordinates represent the point marked B in fig. 48.

(2) *Five holes are equally spaced round a pitch circle of diameter* 10 *in. as shown in fig.* 49. *Calculate the co-ordinates of the holes.*

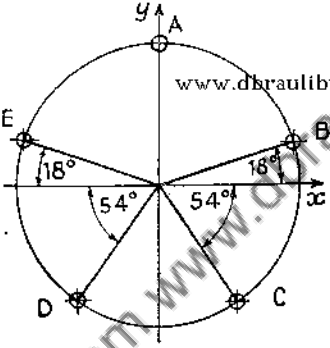


Fig. 49

Hole A. $x = 0 \qquad = y\ \underline{5\text{ in.}}$

Hole B. $x = 5\cos 18° \quad = 5 \times 0{\cdot}9511 = \underline{4{\cdot}756\text{ in.}}$

$y = 5\sin 18° \quad = 5 \times 0{\cdot}3090 = \underline{1{\cdot}545\text{ in.}}$

Hole C. $x = 5\cos 54° \quad = 5 \times 0{\cdot}5878 = \underline{2{\cdot}939\text{ in.}}$

$y = -5\sin 54° \ = -5 \times 0{\cdot}8090 = \underline{-4{\cdot}045\text{ in.}}$

Hole D. $x = -5\cos 54° = \underline{-2{\cdot}939\text{ in.}}$

$y = -5\sin 54° = \underline{-4{\cdot}045\text{ in.}}$

Hole E. $x = -5\cos 18° = \underline{-4{\cdot}756\text{ in.}}$

$y = 5\sin 18° \quad = \underline{1{\cdot}545\text{ in.}}$

(3) *Calculate the co-ordinates of the points A, B and C in fig. 50.*

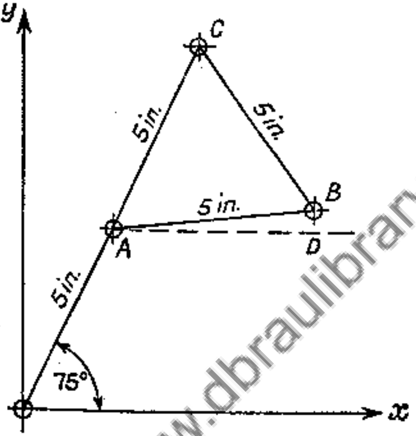


Fig. 50

A. $x = 5 \cos 75° = 5 \times 0{\cdot}2588 = \underline{1{\cdot}294 \text{ in.}}$

$y = 5 \sin 75° = 5 \times 0{\cdot}9659 = \underline{4{\cdot}830 \text{ in.}}$

Since $\angle CAB = 60°$ $\quad \angle BAD = 15°$

B. $x = 1{\cdot}294 + 5 \cos 15° = 1{\cdot}294 + (5 \times 0{\cdot}9659)$
$= 1{\cdot}294 + 4{\cdot}830 = \underline{6{\cdot}124 \text{ in.}}$

$y = 4{\cdot}830 + 5 \sin 15° = 4{\cdot}830 + (5 \times 0{\cdot}2588)$
$= 4{\cdot}830 + 1{\cdot}294 = \underline{6{\cdot}124 \text{ in.}}$

C. Since *OAB* is a straight line $x = 10 \cos 75° = \underline{2{\cdot}588 \text{ in.}}$

$y = 10 \sin 75° = \underline{9{\cdot}660 \text{ in.}}$

(4) *Four holes are equally spaced round a pitch circle as shown in fig.* 51. *Calculate the co-ordinates of the holes.*

Hole A. $x = 5 - 4 \cos 25° = 5 - (4 \times 0{\cdot}9063°) = 5 - 3{\cdot}6252$
$= \underline{1{\cdot}3748 \text{ in.}}$

$y = 4 \sin 25° = 4 \times 0{\cdot}4226 = \underline{1{\cdot}6904 \text{ in.}}$

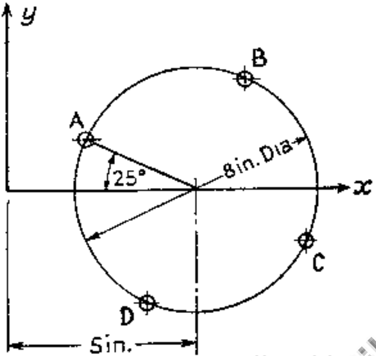


Fig. 51

Hole B. $x = 5 + 4 \cos 65° = 5 + (4 \times 0{\cdot}4226) = 5 + 1{\cdot}6904$
$= \underline{6{\cdot}6904 \text{ in.}}$

$y = 4 \sin 65° = 4 \times 0{\cdot}9063 = \underline{1{\cdot}3748 \text{ in.}}$

Hole C. $x = 5 + 4 \cos 25° = 5 + 3{\cdot}6252 = \underline{8{\cdot}6252 \text{ in.}}$

$y = -4 \sin 25° = -\underline{1{\cdot}6904 \text{ in.}}$

Hole D. $x = 5 - 4 \cos 65° = 5 - 1{\cdot}6904 = \underline{3{\cdot}3096 \text{ in.}}$

$y = -4 \sin 65° = -\underline{3{\cdot}6252 \text{ in.}}$

37. Similar Figures. Similar Solids

(1) *If BC is parallel to XY and AC* = 4·5 *in., BC* = 5·5 *in.,*

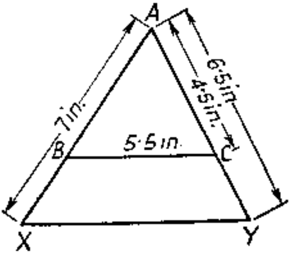


Fig. 52

$AY = 6{\cdot}5$ in., and $AX = 7{\cdot}0$ in., calculate the lengths of XY and AB (see fig. 52).

Since BC is parallel to XY $\triangle ABC$ is similar to $\triangle AXY$ and therefore the ratios of their corresponding sides are equal.

$$\frac{AB}{AX} = \frac{BC}{XY} = \frac{AC}{AY}$$

$$\therefore \frac{AB}{7{\cdot}0} = \frac{5{\cdot}5}{XY} = \frac{4{\cdot}5}{6{\cdot}5}$$

$$\frac{5{\cdot}5}{XY} = \frac{4{\cdot}5}{6{\cdot}5}$$

$$XY = \frac{5{\cdot}5 \times 6{\cdot}5}{4{\cdot}5} = \underline{7{\cdot}95 \text{ in.}}$$

Also
$$\frac{AB}{7{\cdot}0} = \frac{4{\cdot}5}{6{\cdot}5}$$

$$AB = \frac{4{\cdot}5 \times 7{\cdot}0}{6{\cdot}5} = \underline{4{\cdot}85 \text{ in.}}$$

(2) *The tank shown in fig. 53 is filled to $\frac{3}{5}$ of its height. Calculate the dimension marked x.*

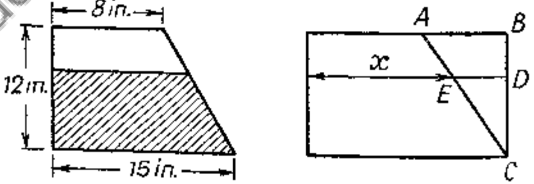


Fig. 53

$$AB = 7 \text{ in.}$$

$$\triangle ABC \text{ is similar to } \triangle EDC$$

$$\therefore \frac{ED}{AB} = \frac{DC}{BC}$$

$$\frac{ED}{7} = \frac{3}{5}$$

$$ED = 4{\cdot}2 \text{ in.}$$

$$\therefore x = 15 - 4{\cdot}2 = \underline{10{\cdot}8 \text{ in.}}$$

(3) *If the ratio of the height of two similar cylinders is 7·5/1, calculate: (a) the ratio of the radii of their bases; (b) the ratio of their cross-sectional areas; (c) the ratio of their volumes.*

(*a*) ratio of radii = $\underline{7{\cdot}5/1}$

(*b*) ∴ ratio of cross-sectional areas $= \dfrac{(7{\cdot}5)^2}{1} = \underline{\dfrac{56{\cdot}25}{1}}$

(*c*) ratio of volumes $= \dfrac{(7{\cdot}5)^3}{1} = \underline{\dfrac{422}{1}}$

NOTE. Areas of similar figures are proportional to the squares of the ratio of corresponding sides.

Volumes of similar solids are proportional to the cubes of the ratio of corresponding sides.

(4) *Calculate the area of a regular hexagon whose sides are 2·5 times those of a similar hexagon of area 6 in.*^{2}

Ratio of length $= \dfrac{2{\cdot}5}{1}$

∴ Ratio of areas $= \dfrac{2{\cdot}5^2}{1} = \dfrac{6{\cdot}25}{1}$

∴ Area of large hexagon $= 6 \times 6{\cdot}25 = \underline{37{\cdot}5 \text{ in.}^2}$

(5) *The ratio of the capacities of two similar tanks is 20 to 1. If the larger tank is 2 ft long, calculate the length of the smaller.*

$$\text{Ratio of capacities} = 20/1$$

$$\text{Ratio of lengths} = \sqrt[3]{\frac{20}{1}}$$

$$= 2{\cdot}71/1$$

$$\text{Length of smaller tank} = \frac{24}{2{\cdot}71} = \underline{8{\cdot}85\text{ in.}}$$

38. Areas: Triangles and Quadrilaterals

NOTE. s rule for area of a triangle.

Let sides of triangle be a, b and c

$$\text{If } s = \tfrac{1}{2}(a + b + c)$$
$$\text{Area} = \sqrt{\{s(s - a)(s - b)(s - c)\}}$$

(1) *Calculate the area of a triangle having sides* 5·5, 6·5 *and* 8 *in.*

$$s = \tfrac{1}{2}\text{ perimeter of triangle}$$
$$= \tfrac{1}{2}(5{\cdot}5 + 6{\cdot}5 + 8) = \tfrac{1}{2} \times 20 = 10\text{ in.}$$
$$\text{Area} = \sqrt{\{s(s - a)(s - b)(s - c)\}}$$
$$= \sqrt{(10 \times 4{\cdot}5 \times 3{\cdot}5 \times 2)}$$
$$= \underline{17{\cdot}75\text{ in.}^2}$$

(2) *Calculate the cross-sectional area of a regular hexagonal bar, having a distance across the flats of* 2·5 *in.*

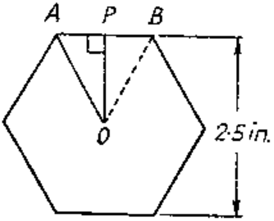


Fig. 54

Referring to fig. 54:

OP is drawn perpendicular to AB. $\angle AOB = 60°$

$$\angle AOP = \tfrac{1}{2}\angle AOB = 30°$$

In $\triangle AOP$

$$PO = 1{\cdot}25 \text{ in.}$$
$$AP = 1{\cdot}25 \tan 30° = 1{\cdot}25 \times 0{\cdot}5774 = 0{\cdot}7218 \text{ in.}$$
$$AB = 0{\cdot}7218 \times 2 = 1{\cdot}4436 \text{ in.}$$

Area of $\triangle AOB$

$= \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times 1{\cdot}4436 \times 1{\cdot}25 = 0{\cdot}9023 \text{ in.}^2$

Area of cross-section

$= 0{\cdot}9023 \times 6 = 5{\cdot}4138 = \underline{5{\cdot}414 \text{ in.}^2}$ correct to 3 decimal places.

(3) *Calculate the area of cross-section of an octagonal bar having sides* 1·5 *in.*

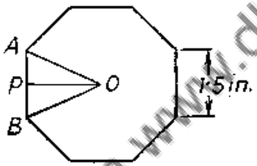


Fig. 55

In fig. 55 OP is perpendicular to AB

$$\angle AOB = \frac{360°}{8} = 45°$$
$$\angle AOP = \tfrac{1}{2}\angle AOB = 22° \, 30'$$
$$AP = 0{\cdot}75 \text{ in.}$$
$$\frac{AP}{PO} = \tan 22° \, 30'$$
$$\therefore \; PO = \frac{AP}{\tan 22° \, 30'} = \frac{0{\cdot}75}{0{\cdot}4142} = 1{\cdot}811 \text{ in.}$$

Area of $\triangle AOP = \frac{1}{2} \times \text{base} \times \text{altitude} = \frac{1}{2} \times 1{\cdot}5 \times 1{\cdot}811$
Area of cross-section $= 8 \times \frac{1}{2} \times 1{\cdot}5 \times 1{\cdot}811 = \underline{10{\cdot}866 \text{ in.}^2}$

(4) *Calculate the dimension x in the trapezium shown in fig. 56 if the area of cross-section is 35 in.*^{2}

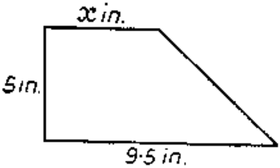


Fig. 56

$$\text{Area of trapezium} = \tfrac{1}{2} \times \text{sum of parallel sides} \times \text{altitude}$$
$$\therefore\ 35 = \tfrac{1}{2} \times (9{\cdot}5 + x) \times 5$$
$$\frac{35 \times 2}{5} = 9{\cdot}5 + x$$
$$14 = 9{\cdot}5 + x$$
$$x = \underline{5{\cdot}5 \text{ in.}}$$

39. Areas: Circles, Sectors and Segments

NOTES. A *sector* of a circle is part of a circle cut off by two radii. A *segment* of a circle is part of a circle cut off by a chord. If this is smaller than a semicircle, it is called a minor segment. If it is larger than a semicircle, it is called a major segment.

In fig. 57(*a*) *AQBO* is a sector and *AQBP* is a minor segment.

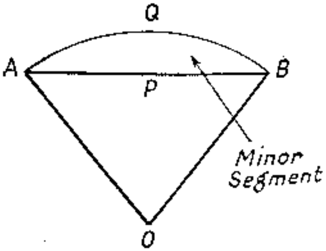
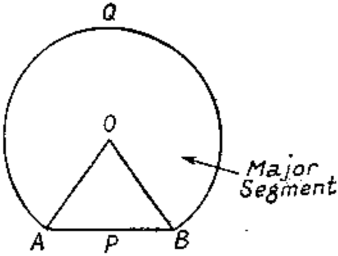


Fig. 57(*a*)



In fig. 57(*b*) *AQBP* is a major segment.

Fig. 57(*b*)

(1) *A tube has an outside diameter of* 1·800 *in. and a wall thickness of* 0·25 *in. Calculate the cross-sectional area.*

$$\text{Inside diameter} = 1{\cdot}800 - (2 \times 0{\cdot}25) = 1{\cdot}300 \text{ in.}$$

$$\text{Area of cross-section} = \frac{\pi \times 1{\cdot}800^2}{4} - \frac{\pi \times 1{\cdot}300^2}{4}$$

$$= \frac{\pi}{4}(1{\cdot}800^2 - 1{\cdot}300^2)$$

$$= \frac{\pi}{4}(3{\cdot}240 - 1{\cdot}690)$$

$$= \frac{\pi}{4} \times 1{\cdot}550$$

$$= \underline{1{\cdot}217 \text{ in.}^2}$$

(2) *Calculate the length of arc and area of a sector of radius* 3·5 *in. and sector angle* 40° 39′.

$$40° \, 39' = 40{\cdot}65°$$

$$\text{Length of arc} = 2\pi r \times \frac{\theta}{360} \text{ where } \theta \text{ is the sector angle.}$$

$$2 \times \pi \times 3{\cdot}5 \times \frac{40{\cdot}65}{360} = \underline{2{\cdot}48 \text{ in.}}$$

$$\text{Area} \quad = \pi r^2 \times \frac{\theta}{360}$$

$$\pi \times 3{\cdot}5^2 \times \frac{40{\cdot}65}{360} = \underline{4{\cdot}34 \text{ in.}^2}$$

(3) *If the area of a sector of a circle of radius 6 in. is to be 45 in.^{2}, calculate the sector angle and the length of the arc of the sector.*

If $\theta°$ is the sector angle, then

$$\pi \times 6^2 \times \frac{\theta}{360} = 45$$

$$\therefore \frac{\pi \times \theta}{10} = 45$$

$$\theta = \frac{45 \times 10}{\pi} = 143{\cdot}2°$$

$$= \underline{143°\ 12'}$$

$$\text{Length of arc} = 2\pi r \times \frac{143{\cdot}2}{360}$$

$$= \frac{2 \times \pi \times 6 \times 143{\cdot}2}{360}$$

$$= \underline{15{\cdot}00\ \text{in.}}$$

(4) *Calculate the area of a segment which subtends an angle of 56° at the centre of a circle of radius 5 in.*

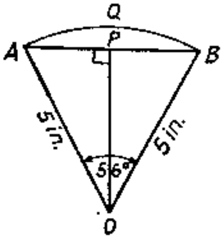


Fig. 58

OP is perpendicular to chord *AB* and bisects the angle *AOB* (fig. 58)

$$\therefore AP = 5 \sin 28° = 5 \times 0{\cdot}4695 = 2{\cdot}348 \text{ in.}$$
$$OP = 5 \cos 28° = 5 \times 0{\cdot}8829 = 4{\cdot}415 \text{ in.}$$

Area of triangle

$$= \tfrac{1}{2} \times AB \times PO = \tfrac{1}{2} \times 4{\cdot}695 \times 4{\cdot}415 = 10{\cdot}36 \text{ in.}^2$$

Area of sector $= \pi \times 5^2 \times \dfrac{56}{360} = 12{\cdot}22 \text{ in.}^2$

Area of segment $= 12{\cdot}22 - 10{\cdot}36 = \underline{1{\cdot}82 \text{ in.}^2}$

(5) *Calculate the area of the segment shown in fig.* 59.

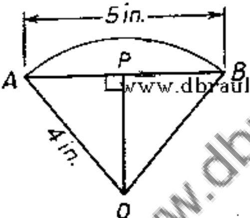


Fig. 59

By Pythagoras' Theorem

$$PO^2 = 4^2 - 2{\cdot}5^2 = 16 - 6{\cdot}25 = 9{\cdot}75$$
$$PO = \sqrt{9{\cdot}75} = 3{\cdot}123 \text{ in.}$$

Area of $\triangle AOB = \tfrac{1}{2} \times 5 \times 3{\cdot}123 = 7{\cdot}81 \text{ in.}^2$

In $\triangle APO$ $\sin\angle AOP = \dfrac{2{\cdot}5}{4} = 0{\cdot}625$

$$\therefore \angle AOP = 38^\circ\, 41'$$
and $\angle AOB = 79^\circ\, 24' = 79{\cdot}4^\circ$

$$\text{Area of sector } AOB = \pi \times 4^2 \times \frac{79{\cdot}4}{360}$$

$$= 11{\cdot}10$$

Area of segment $= 11{\cdot}10 - 7{\cdot}81$
$= \underline{3{\cdot}29 \text{ in.}^2}$

40. Mensuration: Rectangular Blocks, Cylinders and Prisms

FORMULA

The volume of any solid having constant area of cross-section = area of cross section × length, e.g. for cylinder

$$V = \pi d^2 h/4$$

where d is the diameter and h the height (or length).

(1) *Calculate the weight of a cylinder 2·5 in. diameter and 8 in. long if it is made of aluminium weighing* 0·093 *lb/in.*^{3}

$$\text{Volume of cylinder} = \frac{\pi d^2 h}{4} = \frac{\pi \times 2{\cdot}5^2 \times 8}{4} = \frac{\pi \times 6{\cdot}25 \times 8}{4} = 39{\cdot}27 \text{ in.}^3$$

$$\text{Weight} = \text{volume} \times \text{density} = 39{\cdot}27 \times 0{\cdot}093 = \underline{3{\cdot}65 \text{ lb}}$$

(2) *Calculate the length of rectangular strip* $\frac{1}{4} \times \frac{3}{4}$ *in. if it is to be extruded from a cylindrical billet* 2 *ft* 6 *in. long and* 1 *ft diameter.*

$$\text{Volume of cylinder} = \frac{\pi \times 1^2 \times 2{\cdot}5}{4} = 1{\cdot}964 \text{ ft}^3$$

$$\text{Area of cross-section of strip} = \tfrac{1}{4} \times \tfrac{3}{4} = 0{\cdot}1875 \text{ in.}^2 = \frac{0{\cdot}1875}{144} \text{ ft}^2$$

$$\text{Length of strip} = 1{\cdot}964 \div \frac{0{\cdot}1875}{144} = 1{\cdot}964 \times \frac{144}{0{\cdot}1875} = \underline{1509 \text{ ft}}$$

(3) *Calculate the weight per foot length of cylindrical pipe of outside diameter* 3·5 *in. and metal thickness* $\frac{3}{8}$ *in. if it is made of steel weighing* 0·28 *lb/in.*^{3}

$$\text{Sectional area} = \pi \times \frac{3{\cdot}5^2}{4} - \pi \times \frac{2{\cdot}75^2}{4} = \frac{\pi}{4}(3{\cdot}5^2 - 2{\cdot}75^2)$$

$$= \frac{\pi}{4}(12{\cdot}25 - 7{\cdot}56)$$

$$= \frac{\pi}{4} \times 4{\cdot}69$$

$$= 3{\cdot}69 \text{ in.}^2$$

$$\text{Volume} = 3{\cdot}69 \times 12 = 44{\cdot}28 \text{ in.}^3\text{/ft length of pipe}$$

$$\text{Weight} = 44{\cdot}28 \times 0{\cdot}28$$

$$= \underline{12{\cdot}4 \text{ lb/ft}}$$

(4) *Calculate the weight of* 50 *ft of hexagonal bar* 1·25 *in. across the flats if it is made of steel weighing* 0·28 *lb/in.*^{3}

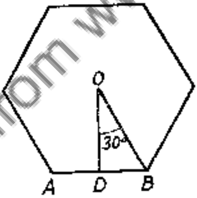


Fig. 60

Referring to fig. 60, *O* is the centre of the bar and *OD* is perpendicular to a side *AB*.

$BD = 0{\cdot}625 \tan 30° = 0{\cdot}625 \times 0{\cdot}5774 = 0{\cdot}3609$ in.
$AB = 2 \times 0{\cdot}3609 \quad = 0{\cdot}7218$ in.

$$\text{Area of } \triangle AOB = \tfrac{1}{2} \times 0{\cdot}625 \times 0{\cdot}7218$$

$$= 0{\cdot}2256 \text{ in.}^2$$

$$\text{Area of cross-section} = 6 \times 0{\cdot}2256$$

$$= 1{\cdot}3536 \text{ in.}^2$$

Volume of 150 ft of bar $= 1{\cdot}3536 \times 150 \times 12$
$= 2436{\cdot}5$ in.³
Weight of bar $= 2436{\cdot}5 \times 0{\cdot}28$
$= \underline{682{\cdot}2 \text{ lb}}$

(5) *Find the weight of the casting shown in fig.* 61 *if it is made of cast iron weighing* 0·26 *lb/in.*³

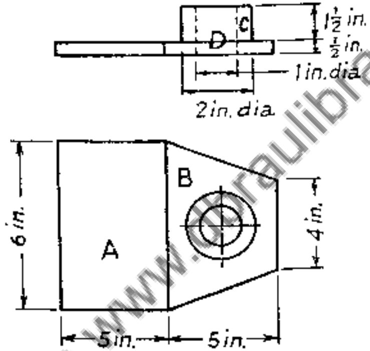


Fig. 61

Volume of A $= 6 \times 5 \times \frac{1}{2} = 15$ in.³
Volume of base-plate $B = \frac{1}{2} \times 5 \times (6 + 4) \times \frac{1}{2} = 12\frac{1}{2}$ in.³
(ignoring hole)

Volume of branch C $= \pi \times \dfrac{2^2}{4} \times 1\frac{1}{2} = 4{\cdot}713$ in.³

Volume of D (hole) $= \dfrac{\pi d^2 h}{4}$

$= \dfrac{\pi \times 1 \times 2}{4} = 1{\cdot}571$ in.³

Volume of casting $= 15 + 12{\cdot}5 + 4{\cdot}713 - 1{\cdot}571$
$= 32{\cdot}213 - 1{\cdot}571$
$= 30{\cdot}642$ in.³

Weight of casting $= 30{\cdot}642 \times 0{\cdot}26$
$= \underline{7{\cdot}97 \text{ lb}}$

41. Mensuration: Pyramids, Cones and Spheres

FORMULAE

Pyramid. Volume = $\frac{1}{3}$ × area of base × altitude.

Cone. Volume = $\frac{1}{3}$ × area of base × altitude (h) = $\frac{1}{3}\pi r^2 h$, where r is the radius of base.

Curved surface area = π × base radius (r) × slant height (l). l, h and r are connected by the formula:

$$l^2 = h^2 + r^2$$

Sphere. Volume = $\frac{1}{6}\pi d^3$ or $\frac{4}{3}\pi r^3$

Surface Area = πd^2 or $4\pi r^2$

where d is the diameter and r the radius.

(1) *A cone has a base diameter of 3 in. and altitude of 4 in. Calculate: (a) its volume; (b) the slant height; (c) curved surface area.*

(*a*) Volume $= \frac{1}{3}\pi r^2 h$

$$\begin{aligned} &= \tfrac{1}{3}\pi \times 1{\cdot}5^2 \times 4 \\ &= \underline{9{\cdot}425 \text{ in.}^3} \end{aligned}$$

(*b*) By Pythagoras' Theorem

$$\begin{aligned} \text{Slant height } l &= \sqrt{(1{\cdot}5^2 + 4^2)} \\ &= \sqrt{(2{\cdot}25 + 16)} \\ &= \sqrt{18{\cdot}25} \\ &= \underline{4{\cdot}272 \text{ in.}} \end{aligned}$$

(*c*) Curved surface area

$$\begin{aligned} &= \pi r l \\ &= \pi 1{\cdot}5 \times 4{\cdot}272 \\ &= \underline{20{\cdot}13 \text{ in.}^2} \end{aligned}$$

(*d*) Total surface area

$$\begin{aligned} &= \text{curved surface area} + \text{area of base.} \\ &= 20{\cdot}13 + \pi \times 1{\cdot}5^2 \\ &= 20{\cdot}13 + 7{\cdot}07 \\ &= \underline{27{\cdot}20 \text{ in.}^2} \end{aligned}$$

(2) *Calculate the weight of* 1 *gross of* $\frac{1}{4}$*-in. diameter ball bearings if they are made of steel weighing* 0·28 *lb/in.*^{3}

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of 1 ball bearing $= \frac{4}{3}\pi \times 0{\cdot}125^3$

$= 0{\cdot}008179$ in.^{3}

Volume of 1 gross ball bearings $= 0{\cdot}008179 \times 144$

$= 1{\cdot}178$ in.^{3}

Weight $= 1{\cdot}178 \times 0{\cdot}28$

$= \underline{0{\cdot}33 \text{ lb}}$

42. Areas of Irregular Figures

The areas of figures which cannot be easily divided into simple components such as a rectangle, triangle or circle can be found using the mid-ordinate rule.

MID-ORDINATE RULE (fig. 62)

Divide the base line into a suitable number of equal intervals of width (d). Usually 6 to 10 intervals are adequate. Measure the height of the mid-ordinate of each section, i.e. the height of the

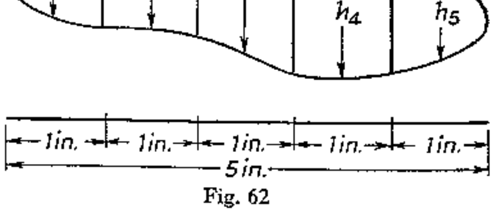


Fig. 62

section at the middle of the interval. Let these heights be h_1, h_2, h_3, etc.

Add together the heights of the mid-ordinates and multiply by the interval width.

$$\text{Area} = d(h_1 + h_2 + h_3 + h_4 + \ldots)$$

Find the weight of the irregular-shaped piece of sheet metal shown in fig. 62, if it is made of sheet weighing 4·5 lb/ft.^{2}

Divide the base line into 5 equal divisions of 1 in. each. The heights of the mid-ordinates of each section are:

$$\begin{aligned} h_1 &= 1{\cdot}37 \text{ in.} \\ h_2 &= 2{\cdot}00 \text{ in.} \\ h_3 &= 2{\cdot}00 \text{ in.} \\ h_4 &= 1{\cdot}56 \text{ in.} \\ h_5 &= 0{\cdot}87 \text{ in.} \end{aligned}$$

$$\begin{aligned} \text{Area} &= 1(1{\cdot}37 + 2{\cdot}00 + 2{\cdot}00 + 1{\cdot}56 + 0{\cdot}87) \\ &= 7{\cdot}80 \text{ in.}^2 \end{aligned}$$

$$\text{Weight} = \frac{7{\cdot}80}{144} \times 4{\cdot}5 \text{ lb} = \frac{7{\cdot}80 \times 4{\cdot}5 \times 16}{144} \text{ oz}$$

$$= \underline{3{\cdot}9 \text{ oz}}$$

43. Graphs from Formulae

(1) *If 1 h.p. = 746 watts (W), construct a graph for converting horse-power to watts for a range of 0–5 h.p. Use the graph (fig. 63) to find (a) the equivalent of 1·25 h.p. in watts; (b) the equivalent of 1000 watts in h.p.*

If
1 h.p. = 746 W
2 h.p. = 746 × 2 = 1492 W
3 h.p. = 746 × 3 = 2238 W
4 h.p. = 746 × 4 = 2984 W
5 h.p. = 746 × 5 = 3730 W

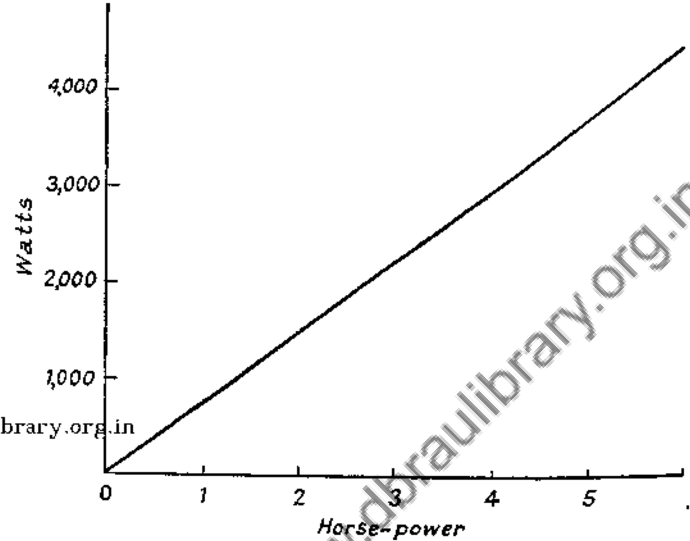


Fig. 63

From graph

1·25 h.p. = 930 W
1000 W = 1·34 h.p.

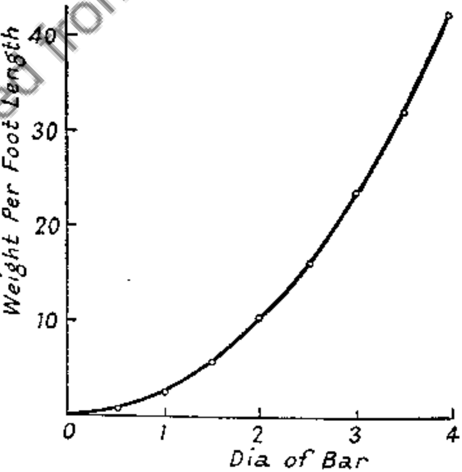


Fig. 64

(2) *The weight per foot length of steel bar is given by the formula* $W = 3\pi\sigma d^2$ *where* d *is the diameter in inches.*

If $\sigma = 0{\cdot}28$, *calculate the weight of bars from* 0 *to* 4*-in. diameter and plot a graph. From it find the weight of* $3\frac{1}{4}$*-in. diameter bar.*

The table of values is:

d	0	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0
W	0	0·66	2·64	5·93	10·55	16·47	23·71	32·20	42·10

We now draw the graph (fig. 64).

From graph. For $3\frac{1}{4}$-in. diameter bar, weight = 27·5 lb

44. Indices

Rules for using indices:

(i) When multiplying, add the indices, e.g.

$$x^2 \times x^3 = (x \times x) \times (x \times x \times x) = x^5$$

(ii) when dividing, subtract the indices, e.g.

$$\frac{x^6}{x^2} = \frac{\cancel{x} \times \cancel{x} \times x \times x \times x \times x}{\cancel{x} \times \cancel{x}} = x^4$$

(iii) $\dfrac{x^6}{x^6} = x^0$ by rule for subtracting indices. But a number divided by itself is equal to 1, hence

$$x^0 = 1$$

(iv) A negative index means divide, e.g.

$$x^{-3} \text{ means } \frac{1}{x^3}$$

(v) A fractional index means a root, e.g.

$x^{1/3}$ means the cube root of x.

$x^{2/3}$ means the square of the cube root of x
or the cube root of the square of x.

(1) *Write down the values of:* (a) 10^3; (b) 10^{-2}; (c) $10^{1/3}$.

(a) $10^3 = 10 \times 10 \times 10 = \underline{1000}$

(b) $10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \underline{\frac{1}{100}}$

(c) $10^{1/3} = \sqrt[3]{10} = \underline{2{\cdot}115}$

(2) *Simplify:* (a) $10^3 \times 10^5 \times 10^6$; (b) $\frac{10^8}{10^3}$; (c) $\frac{10^3}{10^8}$; (d) $\frac{10^5}{10^5}$; (e) $a^4 \times a$; (f) $\frac{x^6}{x^2}$; (g) $\frac{p^2q^5}{p^3q^2}$.

(a) $10^3 \times 10^5 \times 10^6 = \underline{10^{14}}$

(b) $\frac{10^8}{10^3} = \underline{10^5}$

(c) $\frac{10^3}{10^8} = 10^{-5} = \underline{\frac{1}{10^5}}$

(d) $\frac{10^5}{10^5} = 10^\circ = \underline{1}$

(e) $a^4 \times a = \underline{a^5}$

(f) $\frac{x^6}{x^2} = \underline{x^4}$

(g) $\frac{p^2q^5}{p^3q^2} = p^{-1}q^3 = \underline{q^3/p}$

(3) *The maximum distance L ft between bearings of a shaft is given by $L = 8{\cdot}96d^{2/3}$ where d is the diameter of the shaft in inches. Calculate L when $d = 2{\cdot}75$ in.*

$$L = 8{\cdot}96 \times 2{\cdot}75^{2/3}$$

The calculation is carried out as follows: Look up log 2·75 and multiply it by $\frac{2}{3}$, add log 8·96. The antilog of this gives L.

No.	Log
2·75	0·4393
	2
	0·8786
÷ 3	0·2929
8·96	0·9523
	1·2452

$$L = \text{antilog } 1{\cdot}2452 = \underline{17{\cdot}59 \text{ ft}}$$

(4) *The formula* $v = \dfrac{164}{t^{1/7}}$ *gives the connexion between the cutting speed in ft/min and the life in minutes between regrinds of a cutting tool. Calculate* v *if* $t = 200$ *min.*

$$v = \frac{164}{200^{1/7}}$$

The calculation is carried out as follows: divide log 200 by 7, subtract from log 164. The antilog of this gives v.

No.	Log
200	2·3010
÷ 7	0·3287
164	2·2148
	0·3287
	1·8861

$$v = \underline{76{\cdot}93 \text{ ft/min}}$$

45. Transposition of Formulae I

(1) *Transpose the following formulae:* (*a*) $A = LB$ for B; (*b*) $v^2 = 2gh$ *for* g; (*c*) $f = \dfrac{M}{Z}$ *for* Z; (*d*) $H = \dfrac{EI}{746}$ *for* I.

It is helpful for beginners to insert numbers in place of all the symbols except the one to be transposed, this providing a useful hint for transposing with symbols.

(*a*) If $24 = 6\ B$, then $B = \frac{24}{6} = 4$

If $A = LB$, then $\underline{B = \frac{A}{L}}$

(*b*) $v^2 = 2gh$, then $\underline{g = \frac{v^2}{2h}}$

(*c*) Again comparing with a simple equation

$$5 = \frac{15}{Z} \qquad \therefore 5Z = 15 \qquad \therefore Z = \frac{15}{5} = 3$$

If $f = \frac{M}{Z}$, then $fZ = M$ and $\underline{Z = \frac{M}{f}}$

(*d*) $$H = \frac{EI}{746} \qquad \therefore 746H = EI \text{ and } \underline{I = \frac{746H}{E}}$$

(2) *Transpose the following formulae:* (*a*) $V = \frac{t}{2{\cdot}71p}$ *for p;* (*b*) $d^2l = \frac{cK}{N}$ *for c;* (*c*) $H = \frac{d^2Sn}{12{,}720}$ *for S.*

(*a*) $V = \frac{t}{2{\cdot}71p} \qquad \therefore 2{\cdot}71pV = t$ and $\underline{p = \frac{t}{2{\cdot}71V}}$

(*b*) $d^2l = \frac{cK}{N} \qquad \therefore d^2lN = cK$ and $\underline{c = \frac{d^2lN}{K}}$

(*c*) $H = \frac{d^2Sn}{12{,}720} \qquad \therefore 12{,}720H = d^2Sn$ and $\underline{S = \frac{12{,}720H}{d^2n}}$

(3) *The brake horse-power B required to drive a vehicle is given by* $B = \frac{TV}{375e}$. *Transpose the formula to give T and calculate T when* $B = 36$, $e = 0{\cdot}85$ and $V = 45$.

$$B = \frac{TV}{375e}$$

$$\therefore\ 375eB = TV$$

$$\therefore\ T = \frac{375eB}{V}$$

When $B = 36$, $e = 0{\cdot}85$, $V = 45$

$$T = \frac{375 \times 0{\cdot}85 \times 36}{45} = \underline{255}$$

(4) *Transpose the formula* $V = \dfrac{\pi d^2 h}{4}$ *to give* h *and calculate its value when* $V = 330$ *and* $d = 7$.

$$V = \frac{\pi d^2 h}{4}$$

$$\therefore\ 4V = \pi d^2 h$$

$$\therefore\ h = \frac{4V}{\pi d^2}$$

$$= \frac{4 \times 330}{\frac{22}{7} \times 7 \times 7}$$

$$= \frac{4 \times 330}{22 \times 7}$$

$$= \frac{60}{7} = \underline{8{\cdot}6}$$

46. Transposition of Formulae II

(1) *Transpose the formulae:* (*a*) $T = t + 460$ *for* t;
(*b*) $H = S + xL$ *for* S; (*c*) $B = H + 4\pi I$ *for* I.

(*a*) $T = t + 460$ $\quad\therefore\ \underline{t = T - 460}$

(b) $H = S + xL \qquad \therefore\ \underline{S = H - xL}$

(c) $B = H + 4\pi I \qquad \therefore\ 4\pi I = B - H \qquad \therefore\ \underline{I = \dfrac{B - H}{4\pi}}$

(2) *Transpose the formulae:* (a) $1 - \dfrac{A}{L} = n$ *for* A;

(b) $R + \dfrac{r}{n} = S$ *for* r; (c) $C = \dfrac{5}{9}(F - 32)$ *for* F.

(a) $1 - \dfrac{A}{L} = n \qquad \therefore\ L - A = Ln$

$\therefore\ L = Ln + A \qquad \therefore\ \underline{A = L - Ln}$

(b) $R + \dfrac{r}{n} = S \qquad \therefore\ nR + r = nS \qquad \therefore\ \underline{r = nS - nR}$

(c) $C = \dfrac{5}{9}(F - 32) \qquad \therefore\ 9C = 5(F - 32)$

$$9C = 5F - 160$$
$$5F = 9C + 160$$
$$\underline{F = \frac{9C}{5} + 32}$$

(3) *The formula* $D = \dfrac{nd}{13} + 7d$ *is used for wire ropes. Transpose the formula to find* n *and calculate* n *when* $D = 1{\cdot}25$ *and* $d = 0{\cdot}075$.

$$D = \frac{nd}{13} + 7d$$

$$\therefore\ 13D = nd + 91d$$
$$\therefore\ nd = 13D - 91d$$
$$\therefore\ n = \frac{13D - 91d}{d}$$

When $D = 1{\cdot}25$, $d = 0{\cdot}075$

$$n = \frac{13 \times 1{\cdot}25 - 91 \times 0{\cdot}075}{0{\cdot}075}$$

$$n = \frac{16{\cdot}250 - 6{\cdot}825}{0{\cdot}075}$$

$$= \frac{9{\cdot}425}{0{\cdot}075}$$

$$= \underline{126}$$

(4) *The formula* $e = \dfrac{n - a}{m - a}$ *is used in epicyclic trains of gear wheels. Calculate* n *if* $k = 0{\cdot}6$, $a = 24$ *and* $m = 2$.

Transpose the formula to find n

$$e = \frac{n - a}{m - a}$$

$$e(m - a) = n - a$$
$$em - ea = n - a$$
$$n = em - ea + a$$

When $e = 0{\cdot}6$, $a = 24$ and $m = 2$

$$n = 0{\cdot}6 \times 2 - 0{\cdot}6 \times 24 + 24$$
$$= 1{\cdot}2 - 14{\cdot}4 + 24$$
$$= 25{\cdot}2 - 14{\cdot}4$$
$$= \underline{10{\cdot}8}$$

47. Transposition of Formulae III

(1) *Transpose the following formulae:* (*a*) $A = \pi r^2$ *for* r; (*b*) $H = \dfrac{2938}{N^2}$ *for* N; (*c*) $T = \dfrac{wL^2}{8d}$ *for* L

(*a*) $A = \pi r^2 \quad \therefore \quad r^2 = \dfrac{A}{\pi} \quad \therefore \quad \underline{r = \sqrt{\dfrac{A}{\pi}}}$

(*b*) $H = \dfrac{2938}{N^2} \quad \therefore HN^2 = 2938 \quad \therefore N^2 = \dfrac{2938}{H} \quad \therefore \underline{N = \sqrt{\dfrac{2938}{H}}}$

(*c*) $T = \dfrac{wL^2}{8d} \quad \therefore \; 8dT = wL^2 \quad \therefore \; L^2 = \dfrac{8dT}{w} \quad \therefore \; \underline{L = \sqrt{\dfrac{8dT}{w}}}$

(2) *Transpose the following formulae:* (*a*) $B = \sqrt{\frac{2RT}{3}}$ *for* R; (*b*) $T = \sqrt{\frac{P}{2}} - 1$ *for* P; (*c*) $x = \sqrt{\left\{\frac{a(l+b)}{3}\right\}}$ *for* l.

$$(a)\ B = \sqrt{\frac{2RT}{3}} \qquad \therefore\ B^2 = \frac{2RT}{3}$$

$$\therefore\ 3B^2 = 2RT \qquad \therefore\ \underline{R = \frac{3B^2}{2T}}$$

$$(b)\ T = \sqrt{\frac{P}{2}} - 1 \qquad \therefore\ T + 1 = \sqrt{\frac{P}{2}}$$

$$\therefore\ \frac{P}{2} = (T+1)^2 \qquad \therefore\ \underline{P = 2(T+1)^2}$$

$$(c)\ x = \sqrt{\left\{\frac{a(l+b)}{3}\right\}} \qquad \therefore\ x^2 = \frac{a(l+b)}{3} \qquad \therefore\ 3x^2 = a(l+b)$$

$$\therefore\ 3x^2 = al + ab \qquad \therefore\ al = 3x^2 - ab$$

$$\underline{l = \frac{3x^2 - ab}{a}}$$

(3) *The thickness T for the top plate of a semi-elliptical laminated spring is given by the formula* $T = \frac{FL^2}{4{\cdot}3Ed}$. *Calculate L when* $T = \frac{3}{8}$; $F = 60{,}000$; $E = 30{,}000{,}000$ *and* $d = 4$.

$$T = \frac{FL^2}{4{\cdot}3Ed}$$

$$\therefore\ 4{\cdot}3EdT = FL^2$$

$$\therefore\ L^2 = \frac{4{\cdot}3EdT}{F}$$

$$\therefore\ L = \sqrt{\left(\frac{4{\cdot}3EdT}{F}\right)}$$

When $T = \frac{3}{8}$; $F = 60{,}000$; $E = 30{,}000{,}000$ and $d = 4$

$$L = \sqrt{\left(\frac{4{\cdot}3 \times 30{,}000{,}000 \times 4 \times 3}{60{,}000 \times 8}\right)}$$

$$= \sqrt{\left(\frac{4{\cdot}3 \times 30{,}000}{4}\right)}$$

$$= \sqrt{3225} = \underline{56{\cdot}8}$$

(4) *The formula* $T = 0{\cdot}2\ fd^3$ *is connected with shafts under torsion. Transpose the formula to find* d *and calculate* d *when* $T = 50{,}000$ *and* $f = 8000$.

$$T = 0{\cdot}2fd^3$$

$$\therefore\ d^3 = \frac{T}{0{\cdot}2f}$$

$$\therefore\ d = \sqrt[3]{\left(\frac{T}{0{\cdot}2f}\right)}$$

If $T = 50{,}000$ and $f = 8000$

$$d = \sqrt[3]{\left(\frac{50{,}000}{0{\cdot}2 \times 8000}\right)}$$

$$= \sqrt[3]{\frac{50{,}000}{1600}}$$

$$= \sqrt[3]{31{\cdot}25}$$

$$= \underline{3{\cdot}15}$$

48. Moments of Forces

The moment of a force measures its turning effect and it is obtained by multiplying the force by its perpendicular distance from the pivot or fulcrum.

(1) *In fig.* 65 *AB represents a rod, whose weight we can ignore, balancing about a pivot at F. Find the weight W and the force at the pivot.*

Anticlockwise moment about F (i.e. the moment tending to turn the rod in an anticlockwise direction) $= 14 \times 2 = 28$ lb ft.

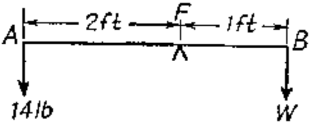


Fig. 65

Clockwise moment about $F = W \times 1$ lb ft.

If the rod balances the two moments are equal, that is, $W \times 1 = 28$

$$W = \underline{28 \text{ lb}}$$

The force at the pivot supports the total load $= 28 + 14 = \underline{42 \text{ lb}}$

(2) *In fig. 66 calculate the value of W if the rod is to balance.*

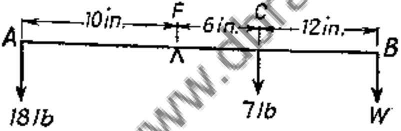


Fig. 66

Anticlockwise moment about $F = 18 \times 10 = 180$ lb in.

Clockwise moment $= (7 \times 6) + (W \times 18)$

$= 42 + 18\,W$ lb in.

The rod balances if

$$180 = 42 + 18\,W$$

$$18\,W = 180 - 42$$

$$= 138$$

$$W = 138/18 = \underline{7\tfrac{2}{3} \text{ lb}}$$

CENTRE OF GRAVITY

Each particle of an object is attracted towards the centre of the earth and we can find a point about which the sum of the moments of the attractions will balance. This point is called the centre of gravity of the object. Sometimes it is obvious where the centre of gravity is, e.g. for a uniform rod the centre of gravity is at its mid point; for a uniform circular disc the centre of gravity is at the centre.

(3) *A tapering pole AB weighing* 30 *lb is* 10 *ft long and its centre of gravity is* 4 *ft from A. If the pole is lying horizontally on the ground what vertical force must be applied at B to lift the end?*

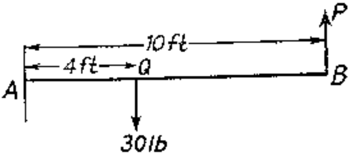


Fig. 67

Let P be the force needed to lift the rod. If the end B is lifted then the pole will pivot about A (fig. 67).

Taking moments about A

$$30 \times 4 = P \times 10$$

$$\underline{P = 12 \text{ lb}}$$

(4) *A uniform girder AB* 30 *ft long weighs* 900 *lb. It is supported* 4 *ft from A and* 8 *ft from B. What is the thrust on each support?*

The 900 lb can be supposed to be concentrated at D the mid point of the rod (fig. 68).

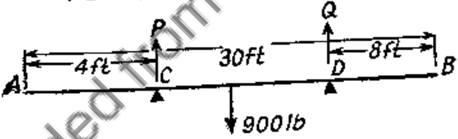


Fig. 68

Here we have two unknown forces. By taking moments about the point of application of one of these forces, say D, we can eliminate Q because the moment of Q about D is zero.

Take moments about D.

$$P \times 18 = 900 \times 7$$

$$P = \frac{900 \times 7}{18} = \underline{350 \text{ lb}}$$

Since $P + Q = 900$

$$Q = 900 - 350 = \underline{550 \text{ lb}}$$

49. Resolution of Forces

(1) *A trolley is pulled along by a rope inclined at an angle of 40° with the horizontal. If the pull on the rope is 100 lb, what is the horizontal force pulling the trolley along and what is the force tending to lift the trolley? If the trolley weighs 150 lb what is the reaction between the trolley and the ground?*

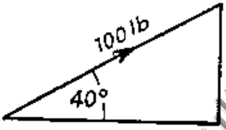


Fig. 69

The effective force pulling the trolley along is the resolved part of the 100 lb in a horizontal direction which is (*see* fig. 69):

$$100 \cos 40° = \underline{76{\cdot}6 \text{ lb}}$$

The resolved part of the 100 lb vertically, i.e.

$$100 \sin 40° = \underline{64{\cdot}3 \text{ lb}},$$

is tending to lift the trolley off the ground. Because of this upward force, the ground does not have to support the whole weight of the trolley but only $150 - 64{\cdot}3 = \underline{85{\cdot}7}$ lb which is thus the reaction between the trolley and ground.

(2) *A tangential force of 220 lb and an inward force of 60 lb are required when cutting a given piece of work. Calculate the resultant force and the angle it makes with the tangential force.*

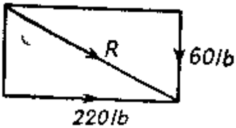


Fig. 70

The resultant force R is given by the diagonal of the rectangle whose sides represent the two given forces (*see* fig. 70).

By Pythagoras' Theorem

$$\begin{aligned} R^2 &= 220^2 + 60^2 \\ &= 48{,}400 + 3600 \\ &= 52{,}000 \\ R &= \sqrt{52{,}000} \\ &= \underline{228 \text{ lb}} \end{aligned}$$

If α is the angle it makes with the tangential force then

$$\tan \alpha = \frac{60}{220} = 0{\cdot}2728$$

$$\underline{\alpha = 15^\circ\ 16'}$$

(3) *An engine weighing* 200 *lb is lifted by a crane. The lifting wire sling is attached symmetrically to the engine at points* 2 *ft apart, and to the crane hook. If each half sling is* 3 *ft long find the tension in each wire* (*see fig*. 71).

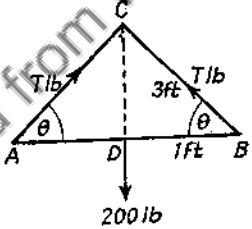


Fig. 71

Since the sling is arranged symmetrically the loads in each half sling are equal to, say T lb. The resolved part of one tension vertically $= T \sin \theta$.

But the resolved parts of both tensions support the 200 lb

$$\therefore\ 2\,T \sin \theta = 200, \text{ or } T = \frac{100}{\sin \theta} \text{ lb}$$

We now have to calculate sin θ. From the triangle BDC, by Pythagoras' Theorem,

$$
\begin{aligned}
CD^2 &= CB^2 - DB^2 \\
&= 3^2 - 1^2 = 8 \\
CD &= 2{\cdot}8284
\end{aligned}
$$

$$\therefore \sin\theta = \frac{CD}{CB} = \frac{2{\cdot}8284}{3} = 0{\cdot}9428$$

$$\therefore T = \frac{100}{0{\cdot}9428} = \underline{106{\cdot}1 \text{ lb}}$$

(4) *A shaft is supported on two bearings A and B 10 ft apart. There is a vertical load of 200 lb 4 ft from A and a horizontal force of 100 lb at the middle point of the shaft. Calculate the magnitude and direction of the resultant force acting on the pivot A (see fig. 72).*

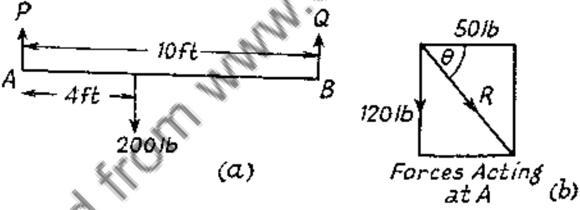


Fig. 72

Due to the horizontal load of 100 lb at the middle point, there will be horizontal loads of 50 lb each at the pivots A and B.

If the vertical reaction at A is P lb, then by taking moments about B

$$
\begin{aligned}
10P &= 200 \times 6 \\
P &= 120
\end{aligned}
$$

At A by Pythagoras' Theorem

$$
\begin{aligned}
R^2 &= 50^2 + 120^2 \\
&= 2500 + 14{,}400 \\
&= 16{,}900 \\
R &= \sqrt{16{,}900} = \underline{130 \text{ lb}}
\end{aligned}
$$

If θ is the angle that the resultant makes with the horizontal

$$\tan\theta = \frac{120}{50} = 2{\cdot}4$$

$$\underline{\theta = 67^\circ\ 23'.}$$

50. Stress

(1) *A steel brake-rod is $\frac{3}{8}$ in. in diameter. If there is a tensile load of* 4,000 *lb in the rod, calculate the stress in tons/in.*^{2}

The cross-sectional area of rod $= \pi \times \frac{1}{4} \times (\frac{3}{8})^2$

$$= 0{\cdot}110 \text{ in.}^2$$

$$\text{Stress} = \frac{\text{load}}{\text{area}} = \frac{4000}{0{\cdot}110} \text{ lb/in.}^2$$

$$= \frac{4000}{2240 \times 0{\cdot}110} \text{ tons/in.}^2$$

$$= \underline{16{\cdot}2 \text{ tons/in.}^2}$$

(2) *A steel rod has to carry a load of* 5 *tons. If the maximum permissible stress is* 16 *tons/in.*^{2}*, calculate a diameter suitable for the rod.*

The minimum cross-sectional area necessary to support load $= \frac{5}{16}$ in.^{2}

If d is the diameter of the rod then

$$\frac{22}{7} \times \frac{d^2}{4} = \frac{5}{16}$$

Cross multiplying

$$22 \times 16 \times d^2 = 5 \times 7 \times 4$$

$$d^2 = \frac{5 \times 7 \times 4}{22 \times 16} = 0{\cdot}398$$

$$d = \sqrt{0{\cdot}398} = \underline{0{\cdot}631 \text{ in.}}$$

The diameter required is a little more than $\frac{5}{8}$ in. and consequently the next available size *larger* than this should be used.

(3) *The force required to shear a bar $1\frac{3}{4}$ in. × $\frac{5}{8}$ in. is 23 tons. Calculate the shear strength of the material.*

Cross-sectional area of bar $= 1\frac{3}{4} \times \frac{5}{8}$

$$= \frac{7}{4} \times \frac{5}{8} = \frac{35}{32} \text{ in.}^2$$

$$\text{Shear strength} = \frac{\text{load}}{\text{area}} = 23 \div \frac{35}{32}$$

$$= 23 \times \frac{32}{35} = \underline{21 \text{ tons/in.}^2}$$

(4) *Calculate the force required in lb to punch a hole $\frac{7}{8}$ in. in diameter through a plate $\frac{3}{16}$ in. thick if the shear strength of the material is 25,000 lb/in.²*

Area of metal to be sheared

= circumference of hole × thickness

$$= \frac{22}{7} \times \frac{7}{8} \times \frac{3}{16} = \frac{33}{64} \text{ in.}^2$$

$$\text{Since shear stress} = \frac{\text{force}}{\text{area}}$$

∴ force = shear stress × area,

$$= 25{,}000 \times \frac{33}{64} = \underline{12{,}900 \text{ lb}}$$

51. Work, Power and Efficiency

(1) *A load of 1 ton is raised by a crane at the rate of 5 ft/sec. Calculate the horse-power required.*

Work done per sec = force × distance = 2240 × 5 ft lb.
1 h.p. = 33,000 ft lb/min = 550 ft lb/sec.

$$\text{Hence h.p. required} = \frac{2240 \times 5}{550} = \underline{20{\cdot}4.}$$

(2) *A planing machine has a stroke of 20 ft. The average force for*

the cutting stroke is 1100 *lb and the stroke takes* 4 *sec. The resistance during the return stroke is* 350 *lb, the stroke taking* $1\frac{3}{4}$ *sec. Calculate:* (*a*) *the average h.p. used in the cutting stroke;* (*b*) *the average h.p. used in the return stroke.*

(*a*) Work done driving the cutting stroke $= 1100 \times 20$ ft lb

Work done per sec. $= \dfrac{1100 \times 20}{4}$

$= 5500$ ft lb

Since 1 h.p. $= 550$ ft lb/sec

h.p. required $= \dfrac{5500}{550} = \underline{10}$

(*b*) Work done during return stroke $= 350 \times 20$ ft lb

Work done per sec $= 350 \times 20 \div 1\frac{3}{4}$

$= \dfrac{350 \times 20 \times 4}{7} = 4000$ ft lb

Therefore h.p. required $= \dfrac{4000}{550} = \underline{7{\cdot}28}$

(3) *The cutting resistance of a lathe tool is* 210 *lb and the average diameter of the work is* 4 *in. If it is revolving at* 220 *rev/min calculate the work done per min in ft lb and the h.p. required.*

The effective distance moved by the force

$$= 4\pi \text{ in./rev}$$
$$= 4\pi \times 220 = 880\pi \text{ in./min}$$
$$= \frac{880\pi}{12} = \frac{220\pi}{3} \text{ ft/min}$$

Work done per min = force × distance

$$= \frac{22}{7} \times \frac{220}{3} \times 210 = 48{,}400 \text{ ft lb}$$

Since 1 h.p. $= 33{,}000$ ft lb/min

then h.p. required $= \dfrac{48{,}400}{33{,}000} = \underline{1{\cdot}47}$

(4) *A load of 480 lb is raised by means of a pulley system whose velocity ratio is 6. If the efficiency is 80 per cent, what force is required?*

If F lb is the required force, work done by this force in moving 6 ft is 6 F ft lb.

The useful work done is to raise 480 lb 1 ft, that is, 480 ft lb.

$$\text{Efficiency} = \frac{\text{Useful work}}{\text{Work put in}}.$$

$$= \frac{480}{6\,F} \times 100 \text{ per cent}$$

but as efficiency is 80 per cent

then
$$\frac{480}{6\,F} \times 100 = 80$$

$$\therefore\ 480 \times 100 = 80 \times 6\,F$$

$$F = \frac{480 \times 100}{80 \times 6} = \underline{100 \text{ lb}}$$

52. Torque

Torque is force × radius

$$\text{Work done per revolution} = 2\pi \times \text{radius} \times \text{force}$$
$$= 2\pi \times \text{torque}$$

(1) *The torque on a shaft is 42 lb in. Calculate the work done in ft lb in 50 revolutions of the shaft.*

$$\text{Work done in 1 revolution} = 2\pi \times \frac{42}{12} \text{ ft lb}$$

$$\text{Work done in 50 revolutions} = 2 \times \frac{22}{7} \times \frac{42}{12} \times 50$$

$$= \underline{1100 \text{ ft lb}}$$

(2) *A wheel is fixed with 6 $\frac{3}{8}$-in. diameter bolts on a pitch circle of diameter 12 in. If the maximum permissible shear stress in the*

bolts is 15 *tons/in.*^{2}, *what is the maximum torque that the wheel can transmit?*

Cross-sectional area of one belt $= \frac{22}{7} \times \frac{1}{4} \times (\frac{3}{8})^2$ in.^{2}

Maximum load bolt can withstand $= \frac{22}{7} \times \frac{1}{4} \times (\frac{3}{8})^2 \times 15 \times 2240$

$$= \frac{7425}{2} \text{ lb}$$

Maximum load that 6 bolts can withstand

$$= 6 \times \frac{7425}{2} = 22{,}275 \text{ lb}$$

Maximum torque that wheel can transmit

$$= 22{,}275 \times \frac{6}{12} \text{ lb ft}$$

$$= \underline{11{,}137{\cdot}5 \text{ lb ft}}$$

(3) *A lathe spindle is driven by a* $1\frac{1}{2}$*-h.p. motor at* 100 *rev/min. If the diameter of the work is* 8 *in., what is the maximum tangential force that can be applied?*

If T lb ft is the torque then work done per min $= 2\pi T \times 100$. But since h.p. is $1\frac{1}{2}$, then work done per min $= 1\frac{1}{2} \times 33{,}000$.

Thus $$2 \times \frac{22}{7} \times T \times 100 = 33{,}000 \times \frac{3}{2}$$

$$\therefore\ 2 \times 22 \times T \times 100 = 7 \times 33{,}000 \times 3$$

$$T = \frac{7 \times 33{,}000 \times 3}{2 \times 22 \times 100} = \frac{315}{4}$$

If F lb is the tangential force, then torque

$$= F \times \frac{4}{12} \text{ lb ft}$$

$$\tfrac{1}{3}F = \frac{315}{4}$$

$$F = \frac{3 \times 315}{4} = \frac{945}{4}$$

$$= \underline{236\tfrac{1}{4} \text{ lb}}$$

(4) *The tension on the driving side of a pulley belt is 200 lb and on the slack side 110 lb. If the diameter of the driving pulley is $1\frac{3}{4}$ ft and it is rotating at 250 rev/min, calculate the h.p. transmitted.*

The radius of the pulley is $\frac{7}{8}$ ft.

Hence torque transmitted $= 200 \times \frac{7}{8} - 110 \times \frac{7}{8}$

$= 90 \times \frac{7}{8}$ lb ft

Work done per min $= (90 \times \frac{7}{8} \times 2\pi) \times 250$

Horse-power $= 90 \times \frac{7}{8} \times 2 \times \frac{22}{7} \times 250/33{,}000$

$= \underline{3\frac{3}{4}}$

53. Friction

(1) *A horizontal force of 8 lb is required to move a block weighing 30 lb resting on a horizontal table. What force would be required if a load of 28 lb were placed on the block?*

$$\text{Coefficient of friction} = \frac{\text{force required to move block}}{\text{normal reaction}}$$

$$= \frac{8}{30} = \frac{4}{15}$$

If a load of 28 lb is put on the block the normal reaction is

$$30 + 28 = 58 \text{ lb.}$$

Force required to move block is

coeff. of friction × normal reaction

$$= \frac{4}{15} \times 58 = \underline{15{\cdot}5 \text{ lb}}$$

(2) *A brake-shoe presses against a brake-drum of diameter* 15 *in. with a force of* 240 *lb. If the coefficient of friction between the lining and the drum is* 0·35 *calculate the braking torque.*

$$\text{Frictional force tangential to drum} = 240 \times 0{\cdot}35$$
$$= 84 \text{ lb}$$

$$\text{Braking torque} = \text{force} \times \text{radius}$$

$$= 84 \times \frac{15}{2} = \underline{630 \text{ lb in.}}$$

(3) *The driving and driven members of a friction clutch have a single belt of contact the mean radius of which is* 8 *in. The coefficient of friction on both sides of the belt is* 0·15. *If the plates are pushed together with a force of* 200 *lb what torque will the clutch transmit?*

The frictional force on each side of the belt

$$= 200 \times 0{\cdot}15 = 30 \text{ lb}$$

Since there are two sides of the belt and the 30 lb force is distributed round a circle of radius 8 in.,

$$\text{torque} = 2 \times 30 \times 8 = \underline{480 \text{ lb in.}}$$

(4) *The table of a planing machine together with a casting weighs* 15 *cwt and there is a downward cutting pressure of* 220 *lb. If the average speed of the machine is* 30 *ft/min while it is cutting, calculate the average horse-power required to overcome friction at the table slides, the coefficient of friction being* 0·1.

$$\text{Total downward thrust} = (15 \times 112) + 220$$
$$= 1680 + 220$$
$$= 1900 \text{ lb}$$

Frictional force to be overcome $= 1900 \times 0{\cdot}1 = 190$ lb.
Work done per minute to overcome frictional forces $= 190 \times 30$

$$\text{Horse-power required} = \frac{190 \times 30}{33{,}000} = \underline{0{\cdot}17}$$

54. Expansion. Specific Heat

NOTE. Increase in the length of a solid when heated (i.e. its expansion) is equal to the original length of the solid × rise in temperature × coefficient of expansion.

Coefficient of expansion of steel	0·0000063 per °F.
Coefficient of expansion of copper	0·0000088 per °F.

(1) *What is the increase in:* (*a*) *the diameter;* (*b*) *the circumference of a steel ring whose diameter is* 3 *in. at* 40° *F if the temperature is raised* 250° *F?*

Rise in temperature $= 250° - 40° = 210°$ F.

(*a*) Expansion in diameter $= 3 \times 210 \times 0{\cdot}0000063$

$$= \underline{0{\cdot}00397 \text{ in.}}$$

(*b*) Original length of circumference $= 3\pi$ in.

Expansion in circumference $3\pi \times 210 \times 0{\cdot}0000063$

$$= \underline{0{\cdot}0125 \text{ in.}}$$

(2) *A copper bar, whose original length at* 20° *F was* 85 *in., lengthens by* 0·135 *when the temperature is raised to* 200° *F. Calculate the coefficient of expansion of copper.*

Rise in temperature $= 200° - 20° = 180°$ F.

$$\text{Coefficient of expansion} = \frac{\text{increase in length}}{\text{original length} \times \text{rise in temperature}}$$

$$= \frac{0{\cdot}135}{85 \times 180}$$

$$= \underline{0{\cdot}0000088}$$

(3) *A bar of copper is* 150·000 *in. long and a bar of steel* 150·054 *in. long at* 60° *F. The bars are heated so that their temperatures remain equal. At what temperature will the bars be the same length?*

Let T be the rise in temperature. For every 1° rise in temperature, the difference in lengths of the bars will decrease by $150 \times (0{\cdot}0000088 - 0{\cdot}0000063)$

$$= 150 \times 0{\cdot}0000025 = 0{\cdot}000375 \text{ in.}$$

$$\therefore\ T = \frac{150{\cdot}054 - 150{\cdot}000}{0{\cdot}000375} = \frac{0{\cdot}054}{0{\cdot}000375} = 144$$

Hence, required temperature = 144° + 60° = 204° F

NOTES. A British Thermal Unit (B.t.u.) is the quantity of heat required to raise the temperature of 1 lb of water 1° F.

A Centigrade Heat Unit (C.h.u.) is the quantity of heat required to raise the temperature of 1 lb of water 1° C.

Specific heat

$$= \frac{\text{heat required to raise temperature of 1 lb of a substance } 1^\circ}{\text{heat required to raise the temperature of 1 lb of water } 1^\circ}$$

(4) *A copper bar of rectangular section 2 in. × $1\frac{1}{2}$ in. is 4 ft long. Calculate the number of heat units required to raise its temperature by 400° F assuming that copper weighs 0·32 lb/in.^{3} and the specific heat of copper* = 0·095.

Volume of copper = $2 \times 1\frac{1}{2} \times 48 = 144$ in.^{3}

Weight of copper = 144 × 0·32 = 46·08 lb.

Heat units required

= weight × rise in temperature × specific heat

= 46·08 × 400 × 0·095

= 1750 B.t.u.

(5) *An oiled-fired furnace raises the temperature of aluminium castings weighing 800 lb by 400° C in an hour, the specific heat of aluminium being 0·21. If the furnace uses 40 lb of oil in the hour and each lb gives 18,000 B.t.u. when burnt, calculate the thermal efficiency of the furnace.*

Since the quantity of heat is measured in B.t.u., the temperature rise must be converted to degrees Fahrenheit.

Now a rise of 5° C = 9° F

$$\text{a rise of } 400^\circ\text{ C} = \frac{400 \times 9}{5} = 720^\circ\text{ F}$$

Quantity of heat taken up by the castings

$$= 800 \times 720 \times 0{\cdot}21 \text{ B.t.u.}$$

Quantity of heat produced by combustion of the oil

$$= 40 \times 18{,}000 \text{ B.t.u.}$$

Hence thermal efficiency of the furnace

$$= \frac{\text{heat taken up by the castings}}{\text{heat supplied by the combustion of the oil}}$$

$$= \frac{800 \times 720 \times 0{\cdot}21}{40 \times 18{,}000} = 0{\cdot}168 \text{ or } 16{\cdot}8 \text{ per cent}$$

(6) *A piece of steel bar weighing* 15 *lb was taken from a furnace and plunged into an oil bath containing* 100 *lb of oil at* 60 *F. If the specific heat of steel is* 0·115 *and that of oil* 0·4, *find the temperature of the steel when taken from the furnace if the final temperature of the oil was* 90° *F.*

If T° F was the temperature of the steel when taken from the furnace then the fall in temperature is $(T - 90)$° F, then quantity of heat lost by the steel $= (T - 90) \times 15 \times 0{\cdot}115$.

Rise in temperature of the oil = 90 − 60 = 30° F, therefore quantity of heat gained by the oil $= 30 \times 100 \times 0{\cdot}4$.

Since the heat lost by the steel equals the heat gained by the oil

$$(T - 90) \times 15 \times 0{\cdot}115 = 30 \times 100 \times 0{\cdot}4$$

$$T - 90 = \frac{30 \times 100 \times 0{\cdot}4}{15 \times 0{\cdot}115} = 696$$

$$T = 90 + 696 = \underline{786° \text{ F}}$$

Miscellaneous Examples

(1) A *non-ferrous alloy A consists of copper and zinc only, and* 10·5 *lb of this alloy contains* 7·35 *lb of copper. Determine the percentage composition of this alloy by weight.*

What additions must be made to 15 *cwt of alloy A to produce* 1 *ton of an alloy B containing* 58 *per cent of copper,* 40 *per cent of zinc and* 2 *per cent of tin?*

Percentage of copper in alloy $A = \frac{7{\cdot}35}{10{\cdot}5} \times 100 = 70$. Hence percentage of zinc = 30.

In 15 cwt of alloy A there is $15 \times \frac{70}{100} = 10{\cdot}5$ cwt of copper and thus $15 - 10{\cdot}5 = 4{\cdot}5$ cwt of zinc.

For alloy B 20 cwt contains 58 per cent of copper

$$\text{Weight of copper} = \frac{20 \times 58}{100} = 11{\cdot}6 \text{ cwt}$$

Also 20 cwt contains 40 per cent zinc.

$$\text{Weight of zinc} = \frac{20 \times 40}{100} = 8{\cdot}0 \text{ cwt}$$

Hence, weight of tin $= 20 - (11{\cdot}6 + 8{\cdot}0) = 0{\cdot}4$ cwt.

Qualities to be added to 15 cwt of alloy A are:

Copper $11{\cdot}6 - 10{\cdot}5 = 2{\cdot}1 \text{ cwt} = \underline{235{\cdot}2 \text{ lb}}$

Zinc $8{\cdot}0 - 4{\cdot}5 = 3{\cdot}5 \text{ cwt} = \underline{448{\cdot}0 \text{ lb}}$

Tin $0{\cdot}4 - 0 = 0{\cdot}4 \text{ cwt} = \underline{44{\cdot}8 \text{ lb}}$

(2) *Cast iron weighs* 0·26 *lb/in.*^{3}, *cast aluminium weighs* 0·093 *lb/in.*^{3} *An iron casting consists of a disc* 10 *in. in diameter and* $\frac{3}{4}$ *in. thick, cored with a hole* 4 *in. in diameter. Find:* (*a*) *the actual saving in weight;* (*b*) *the percentage saving in weight if aluminium is used instead of cast iron.* (*Take* π *as* $3\frac{1}{7}$.)

Cross-sectional area of casting $\frac{\pi}{4} \times 10^2 - \frac{\pi}{4} \times 4^2$

$$= \frac{22}{7} \times \frac{1}{4}(100 - 16)$$

$$= \frac{22}{7} \times \frac{1}{4} \times 84 = 66 \text{ in.}^2$$

Volume of casting $= 66 \times \frac{3}{4} = 49{\cdot}5$ in.^{3}

Weight of iron casting $= 49{\cdot}5 \times 0{\cdot}26 = 12{\cdot}87$ lb

Weight of aluminium casting $= 49{\cdot}5 \times 0{\cdot}093 = 4{\cdot}60$ lb (*a*) Saving in weight $= 12{\cdot}87 - 4{\cdot}60 = \underline{8{\cdot}27 \text{ lb}}$

(*b*) Percentage saving in weight $= \dfrac{8{\cdot}27 \times 100}{12{\cdot}87} = \underline{64}$

(3) *When a rectangular steel bar is simply supported over a span of L in. the depth of the bar being D in. and width B in. the safe distributed load W lb/in. that can be carried is given by*

$$W = \frac{KBD^2}{L^2}$$

It is known that when B = 1 *in., D* = 2 *in. and L* = 1 *ft, W can be* 80 *lb/in.: (a) rearrange the formula to give K in terms of the other symbols; (b) determine K, giving both its numerical value and the unit in which it is stated; (c) determine W, if the same bar is used but flat face down, so that B* = 2 *in. and D* = 1 *in.*

(*a*) $$W = \frac{KBD^2}{L^2}$$

$$WL^2 = KBD^2$$

$$K = \frac{WL^2}{BD^2}$$

(*b*) If $B = 1$, $D = 2$, $L = 12$ and $W = 80$ then

$$K = \frac{80 \times 12 \times 12}{1 \times 2 \times 2} = \underline{2880 \text{ lb/in.}^2}$$

$$\left(\text{For units } K = \frac{\text{lb/in.} \times \text{in.} \times \text{in.}}{\text{in.} \times \text{in.} \times \text{in.}} = \text{lb/in.}^2\right)$$

(*c*) $$W = \frac{2880\, BD^2}{L^2}$$

If $B = 2$, $D = 1$, $L = 12$ then

$$W = \frac{2880 \times 2 \times 1}{12 \times 12} = \underline{40 \text{ lb/in.}}$$

(4) *A gallon of water weighs* 10 *lb. A cubic foot of water weighs* $62\frac{1}{2}$ *lb. A tank of dimensions shown in fig.* 73 *contains water to within*

1 *in. of the top. How much water does it contain in gallons? Give your answer correct to three significant figures.*

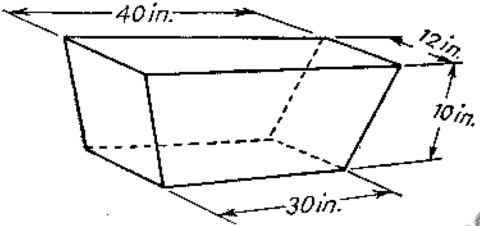


Fig. 73

If water is 9 in. deep then dimensions of surface of water are 39 in. × 12 in.

Area of trapezium (cross-section) $= \frac{9}{2}(39 + 30) = \frac{9 \times 69}{2}$ in.²

$$\text{Volume of water} = 12 \times \frac{9 \times 69}{2} \text{ in.}^3$$

$$= \frac{12 \times 9 \times 69}{12 \times 12 \times 12 \times 2}$$

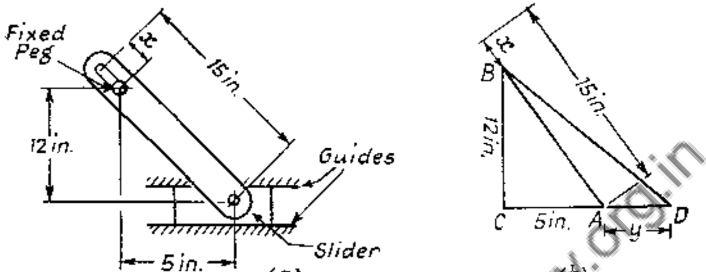
$$= \frac{69}{32} \text{ ft}^3$$

$$\text{Weight of water} = \frac{69}{32} \times 62{\cdot}5 \text{ lb}$$

$$\text{Volume of water} = \frac{69 \times 62{\cdot}5}{32 \times 10} \text{ gal}$$

$$= \underline{13{\cdot}5 \text{ gal}}$$

(5) *In the mechanism shown in fig.* 74(*a*) *the slider moves horizontally in the guides and the travel of the link is limited by a fixed peg. The diagram shows the slider as far to the left as it can travel:* (*a*) *for this position determine by calculation the distance x;* (*b*) *how much can the slider move to the right before it is stopped by the action of the other end of the slot contacting the fixed peg?*



(a) (b)

Fig. 74

In fig. 74(*b*) the slider is represented by *A* and the fixed peg by *B*.

(*a*) By Pythagoras' Theorem $AB^2 = 12^2 + 5^2 = 169$

$$AB = \sqrt{169} = 13$$
$$x = 15 - AB$$
$$= 15 - 13 = \underline{2\text{ in.}}$$

(*b*) The second position is shown as *BD* and $AD = y$ is distance travelled.

In the triangle *BCD*, $BC = 12$ in., $BD = 15$ in.

By Pythagoras' Theorem

$$CD^2 = 15^2 - 12^2 = 81$$
$$CD = 9$$
$$y = CD - 5 = 9 - 5$$
$$= \underline{4\text{ in.}}$$

(6) *Determine by calculation:* (*a*) *the angle A;* (*b*) *the length CD;* (*c*) *the angle B of the template shown in fig.* 75.

The points *M* and *P* have been added to the diagram given in the question.

(*a*) In the triangle *CMD*, $MD = \frac{1}{2} \times 1{\cdot}56 = 0{\cdot}78$

$$\tan A = \frac{CM}{MD} = \frac{1{\cdot}04}{0{\cdot}78} = 1{\cdot}3333$$

$$\underline{A = 53^\circ\, 8'}$$

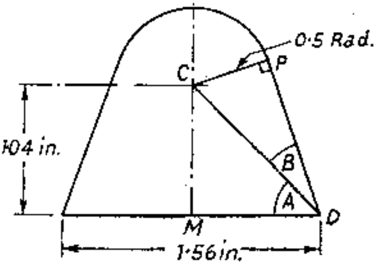


Fig. 75

(*b*) By Pythagoras' Theorem

$$\begin{aligned} CD^2 &= MD^2 + CM^2 \\ &= 0{\cdot}78^2 + 1{\cdot}04^2 \\ &= 0{\cdot}6084 + 1{\cdot}0816 \\ &= 1{\cdot}6900 \\ CD &= \sqrt{1{\cdot}69} = \underline{1{\cdot}3 \text{ in.}} \end{aligned}$$

(*c*) In the triangle CPD, $CP = 0{\cdot}5$, $CD = 1{\cdot}3$

$$\sin B = \frac{CP}{CD} = \frac{0{\cdot}5}{1{\cdot}3} = 0{\cdot}3846$$

$$\underline{B = 22^\circ\ 37'}$$

(7) *The following table gives the weight of steel tube W in lb/ft run, for tube of constant outside diameter 3 in., having various wall thicknesses T in. Plot these values.*

T in.	0·1	0·2	0·3	0·4	0·5
W lb/ft	3·16	5·91	8·55	10·9	13·2

Plot T horizontally, to a scale of 1 in. = 0·1 in. and W vertically, to a scale of 1 in. = 2 lb/ft. Draw a *smooth curve* through the points.

Using your graph, determine the inside diameter of a tube of outside diameter 3 in. weighing 7·26 lb/ft run.

From the graph the inside diameter required = $\underline{0{\cdot}243 \text{ in.}}$

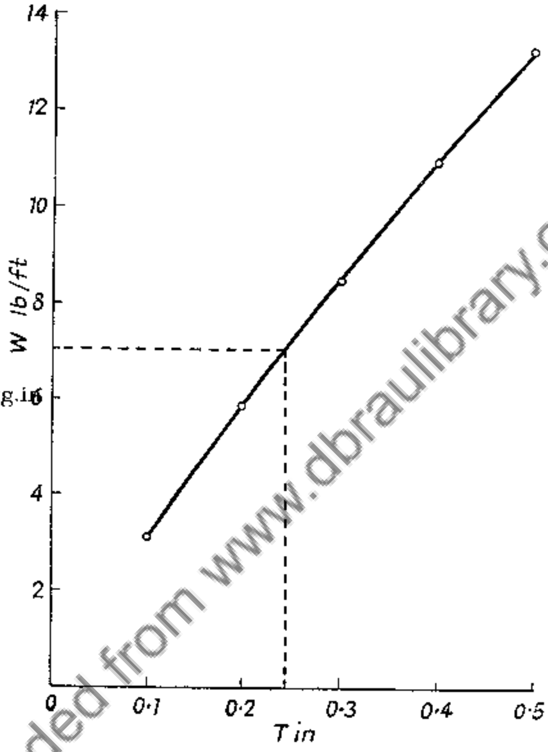


Fig. 76

(8) *If a band is wrapped round a disc and 1½ in. allowed to overlap, the length L (in.) of the band is given by the formula:*

$$L = \pi D + 1{\cdot}5$$

where D is the diameter of the disc in inches. Construct a graph that can be used to determine the values of L for values of D from 5 in. to 10 in.

From your graph, determine the value of D when L is 24 in.

The table of values is first constructed for $L = 5, 6, 7, 8, 9, 10$.

e.g. when $D = 7$ $\quad$ $L = 7 \times 3{\cdot}14 + 1{\cdot}5$
$= 23{\cdot}5$

The complete table of values is

D in.	5	6	7	8	9	10
L in.	17·2	20·3	23·5	26·6	29·8	32·9

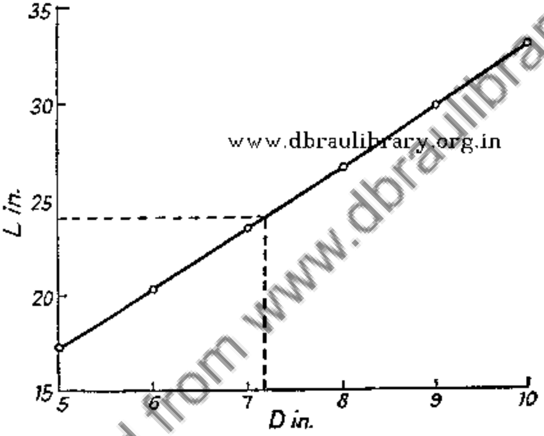


Fig. 77

The graph is found to be a straight line and the value of D when $L = 24$ is read off as indicated in fig. 77 and found to be 7·17 in.

(9) *If the chain shown in fig. 78 is used to lift a load of* $1\frac{1}{2}$ *tons, determine:* (*a*) *the maximum tensile stress in the side plates;* (*b*) *the shearing stress in the pin.*

(*a*) Maximum tensile stress in side plates will be where the width is a minimum. Cross-sectional area of one plate at this place $= \frac{1}{4} \times 1 = \frac{1}{4}$ in.^{2} Each plate carries $\frac{1}{2} \times 1\frac{1}{2} = \frac{3}{4}$ ton.

Maximum tensile stress $= \frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times 4 = \underline{3\text{ tons/in.}^2}$

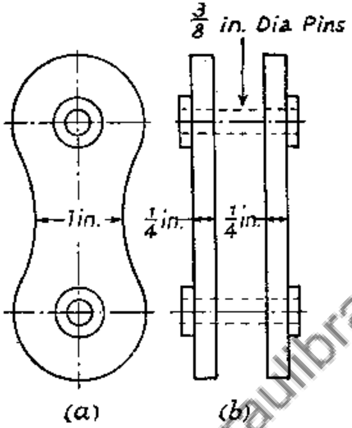


Fig. 78

(*b*) Cross-sectional area of pin $= \frac{\pi}{4} \times (\frac{3}{8})^2$

$$= \frac{22}{7} \times \frac{1}{4} \times \frac{9}{64} = 0{\cdot}110 \text{ in.}^2$$

Load in each side plate = 0·75 ton. Since the bolt is in double shear, the shear stress is

$$\frac{0{\cdot}75}{2 \times 0{\cdot}110}$$
$$= \underline{3{\cdot}4 \text{ tons/in.}^2}$$

(10) *Fig. 79 shows a tie rod subjected to a tensile load of* $2\frac{1}{2}$ *tons:* (*a*) *determine the tensile stress in the shank;* (*b*) *determine the average compressive stress over the underside of the head;* (*c*) *make a sketch to indicate the surface across which there is shearing stress in the head, and calculate the average value of this stress.* (*You may assume* $\frac{1}{\pi} = 0{\cdot}32$.)

(*a*) Cross-sectional area of shank $= \frac{\pi}{4} \times 1 \text{ in.}^2$

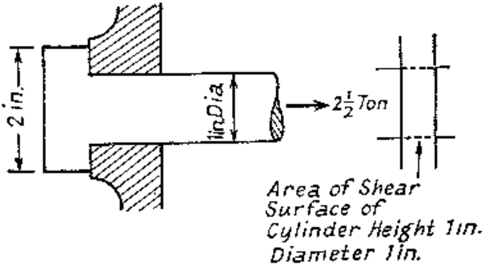


Fig. 79

$$\text{Tensile stress in shank} = 2{\cdot}5 \div \frac{\pi}{4}$$

$$= 2{\cdot}5 \times \frac{4}{\pi}$$

$$= 10 \times 0{\cdot}32$$

$$= \underline{3{\cdot}2 \text{ tons/in.}^2}$$

(*b*) $$\text{Area of compression on head} = \frac{\pi}{4} \times 2^2 - \frac{\pi}{4} \times 1^2$$

$$= \frac{3\pi}{4}$$

$$\text{Compressive stress} = 2{\cdot}5 \div \frac{3\pi}{4} = \frac{2{\cdot}5 \times 4}{3\pi} = \underline{1{\cdot}1 \text{ ton/in.}^2}$$

(*c*) Area across which there is a shearing force in head is

$$\pi \times 1 \times 1 = \pi \text{ in.}^2$$

$$\text{Shear stress} = \frac{2{\cdot}5}{\pi} = 2{\cdot}5 \times 0{\cdot}32 = \underline{0{\cdot}8 \text{ tons/in.}^2}$$

(11) *A certain planing machine can be assumed to have an efficiency of* 80 *per cent when the input is* 2½ *horse-power. Under certain conditions the load due to cutting is* 2200 *lb. The frictional load is* 1100 *lb irrespective of the direction of the stroke. Determine*

the maximum speed of the table: (a) on the cutting stroke; (b) on the return stroke.

$$\text{Output of machine} = 2\tfrac{1}{2} \times 33{,}000 \times \frac{80}{100}$$

$$= 66{,}000 \text{ ft lb/min}$$

(*a*) Total force required in cutting stroke = 2200 + 1100
= 3300 lb

$$\text{Speed of cutting stroke} = \frac{66{,}000}{3300} = \underline{20 \text{ ft/min}}$$

(*b*) Force required on return stroke = 1100 lb

$$\text{Speed of return stroke} = \frac{66{,}000}{1100} = \underline{60 \text{ ft/min}}$$

(12) *During the tapping of a $\frac{5}{8}$-in. Whitworth hole, having* 11 *t.p.i., it is found necessary to apply forces each of* 14 *lb weight at the ends of a double-ended tap wrench of* 18 *in. overall length. If the tap is screwed steadily down by the continued application of such* 14*-lb forces, calculate: (a) the turning movement exerted on the tap; (b) the work done in tapping a one-inch length of thread (assume* $\pi = \frac{22}{7}$*).*

(*a*) Turning movement exerted on the tap = force × distance between forces = 14 × $1\frac{1}{2}$ = 21 lb ft.

(*b*) Work done in one rotation of wrench

$$= 2\pi \times \text{torque}$$
$$= 2\pi \times 21 = 42\pi \text{ ft lb}$$

Number of revolutions of wrench required to tap a 1-in. length of thread is 11.

∴ Work done in tapping 1-in. length of thread

$$= 42 \times \frac{22}{7} \times 11$$

$$= \underline{1452 \text{ ft lb}}$$

(13) *A collar and shaft are made of the same material, having a coefficient of linear expansion of* 0·000006 *per °F. At room temperature of* 60° *F., the shaft is* 5·006 *in. diameter and the collar is* 5·000 *in. diameter. The collar is now heated to* 460° *F.: (a) what is the diametral clearance if the collar is now put on the shaft; (b) assuming that the shaft remains at* 60° *F. while the collar cools, at what temperature will the collar first make contact all round the shaft?*

(*a*) Rise in temperature = 460° − 60° = 400° F.

Increase in diameter of collar = 5·000 × 400 × 0·000006
= 0·012 in.

New diameter of collar = 5 + 0·012 = 5·012 in.
Diametral clearance = 5·012 − 5·006 = 0·006 in.

(*b*) The collar will be in contact all round the shaft when it has contracted 0·006 in., that is, when the temperature has fallen by 200° F. Thus, the required temperature of the collar is 260° F.

(14) *In a heat-treatment shop batches of work are quenched at regular intervals. The heat given to the liquid in the quenching tanks is carried away by an independent water circulation, the rate of flow being* 720 *lb/hr. This cooling water enters at* 60° *F. and leaves* 100° *F., the actual temperature of the quenching liquid remaining at about* 120° *F. It may be assumed that* 80 *per cent of the heat given up by the batches is carried away by the circulating water: (a) what amount of heat in B.t.u./min is carried away by the cooling water? (b) how often are batches of work each weighing* 9 *lb and of specific heat* 0·111 *being quenched from* 1320° *F.?*

(*a*) Rise in temperature of cooling water = 100° − 60° = 40° F.
Flow of water = 720 lb/hr = 12 lb/min.
Quantity of heat carried away by cooling water

= 40 × 12 = 480 B.t.u./min

(*b*) Quantity of heat carried away by cooling water is 80 per cent of heat given up by batches.

$$\therefore \text{ heat given by batches} = \frac{480 \times 100}{80}$$

$$= 600 \text{ B.t.u./min}$$

Fall in temperature of the batches = 1320° − 120° = 1200° F.
Quantity of heat lost in quenching one batch

$$= 1200 \times 0{\cdot}111 \times 9 = 1200 \text{ B.t.u. (approx.)}$$

Hence, a batch is quenched every $\frac{1200}{600} = \underline{2 \text{ min}}$

TO THE READER

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